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PROCEEDING

THE 3RD INTERNATIONAL CONFERENCE ON MATHEMATICS AND STATISTICS

BOGOR, 5 – 6 AUGUST 2008

Mathematics and Statistics: bridge for academia, business, and government in the entrepreneurial era

3rd ICoMS 2008

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Mathematics and Statistics: bridge for academia, business, and government in the entrepreneurial era

organized by

MSMSSEA (Moslems Statisticians and Mathematicians Society in South East Asia)

Department of Statistics
Department of Mathematics
Institut Pertanian Bogor

Department of Mathematics
Universiti Malaysia Terengganu,
Malaysia

PREFACE

Assalaamu'alaikum warahmatullaahi wabarakaatuh

Welcome all participants of ICoMS 2008 to Bogor – Indonesia. This event is organized by MSMSSEA in collaboration with Institut Pertanian Bogor (Indonesia) and Universiti Malaysia Terenganu (Malaysia).

We, the organizing committee, are very glad having this international conference due to many reasons.

1. ICoMS is a good avenue for mathematicians, statisticians, and other scientist to communicate.
2. ICoMS 2008 has a theme related to entrepreneurial era which is very important for mathematicians and statisticians, and scientist in general.
3. The event is important venue for business group, government, and academia to communicate and share knowledge as well.
4. Bogor is beautiful place in Indonesia surrounded by many research centers, IPB, Botanical garden, an other point of interest related to research institution.

We are also happy that the Vice President of Republic of Indonesia, Ministry of National Education, Ministry of Energy and Mineral Resources, and Ministry of Communication and Information Technology are supporting to the ICoMS 2008.

This event held on two days, August 5-6, and consist of several parts. We invite 17 outstanding professors to share and discuss topics in mathematics and statistics, including application. As many as 170 paper and 30 posters presented during this two-day conference. We appreciate to all of contributor from various countries who are motivated to participate in this event.

High appreciation is also awarded to companies and agencies which facilitate so that the even could run well.

We really hope all participants can benefit many things from this international event. May God bless you.

Wa'alaikumsalam warahmatullaahi wabarakaatuh.

The Committee of ICoMS 2008

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ESTIMATION OF OUTSTANDING CLAIMS LIABILITY AND SENSITIVITY ANALYSIS: PROBABILISTIC TREND FAMILY (PTF) MODEL

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Abstract. *Outstanding claims liability is the insurance companies' corporate responsibility towards future debt. There are a number of existing models used to estimate the outstanding claims liability for long tail business. However, some of the assumptions imposed on these models are not met by the claims data under consideration. Furthermore, trends in the development year and payment year periods are not taken into account by these models. The Probabilistic Trend Family (PTF) model is developed to overcome these problems.*
In this paper, PTF model is described; and the process of estimating the outstanding claims liability, starting from the preliminary analysis of the claims data to the estimation of the outstanding claims liability is explained. Furthermore, the sensitivity of the obtained estimate of the outstanding claims liability is analyzed using a tool called leverage. Leverage can be used to evaluate the robustness of the model used to estimate the outstanding claims liability.

***Keywords** : Outstanding claims liability, long tail business, development year period, payment year period , probabilistic trend family, sensitivity, leverage.*

1. Introduction

In general insurance, claims are not usually paid as soon as they occur. A number of factors may cause delay between the occurrence and reporting of a claim and delay between the time of reporting and settlement of a claim. In some cases it may take many years until the final payment. Lines of business with this characteristic of claims payment are called long tailed business.
General insurance companies are required to set aside (reserve) enough of their premium income to cover future claim payments from past and current policies.
Based on claims experience in the past, a claim analyst tries to forecast a value for the amount of the liability contingent to events which have yet to happen. In the past, the objective was to obtain a forecast of the *central* estimate of the distribution of the outstanding claims liability. For many years, actuaries have been applying deterministic forecasting methodologies to estimate the outstanding claims liability. However, in order to quantify the uncertainties on the estimate and to predict the distribution of the outstanding claims liability, the problem needs to be approached stochastically. For the last two decades at least, actuaries and statisticians have been developing statistical forecasting models to estimate the outstanding claims liability as well as measuring its uncertainty.
There are a number of existing models used to estimate the outstanding claims liability for long tail business. However, some of the assumptions imposed on these models are not met by the claims data under consideration. Furthermore, trends in the development year and payment year periods are not taken

into account by these models. The common causes of this failure to satisfy assumption motivated the development of the statistical modeling framework. The rich family of statistical models in the framework contains assumptions more in keeping with reality. This statistical modeling framework is based on the analysis of the logarithms of the incremental data. Each model in the framework has four components of interest. The first three components are trends in each of the directions: development period, accident period, and payment/calendar period, while the fourth component is the distribution of the data about the trends. This rich family of models is often named as the Probabilistic Trend Family (PTF).

2. Probabilistic Trend Family (PTF) Model

The Probabilistic Trend Family (PTF) modelling framework developed by Barnett and Zehnwirth (2000). The Probabilistic Trend Family (PTF) model introduce a modeling framework for the logarithms of the incremental data that allows for changes in trends. The mathematical formulation of the models is given by equation (1)

$$y(i,j) = \alpha_i + \sum_{k=1}^{j} \gamma_k + \sum_{t=1}^{i+j} \iota_t + \varepsilon_{i,j} \quad (1)$$

$$\varepsilon_{i,j} \cong Normal(0, \sigma^2)$$

$$y(i,j) = \ln(p_{i,j})$$

$$y(i,j) \sim \text{Normal}\left(\alpha_i + \sum_{k=1}^{j} \gamma_k + \sum_{t=1}^{i+j} \iota_t , \sigma^2 \right)$$

$$E(y(i,j) \mid i,j) = \left(\alpha_i + \sum_{k=1}^{j} \gamma_k + \sum_{t=1}^{i+j} \iota_t \right) \quad (2)$$

$$Var(y(i,j) \mid i,j) = \sigma^2$$

$$p_{i,j} \sim \text{Lognormal}\left(\alpha_i + \sum_{k=1}^{j} \gamma_k + \sum_{t=1}^{i+j} \iota_t , \sigma^2 \right)$$

$$E(p_{i,j} \mid i,j) = \exp\left(\alpha_i + \sum_{k=1}^{j} \gamma_k + \sum_{t=1}^{i+j} \iota_t + \frac{1}{2}\sigma^2 \right)$$

$$Var(p_{i,j} \mid i,j) = \left[\exp\left(\alpha_i + \sum_{k=1}^{j} \gamma_k + \sum_{t=1}^{i+j} \iota_t + \frac{1}{2}\sigma^2 \right) \sqrt{\exp(\sigma^2) - 1} \right]^2$$

$p_{i,j}$: *incremental payment data for accident period i and development period j*

$y(i,j)$: *natural* log *of the incremental payment data in accident period i and development period j*

α_i : *level for accident period i*

γ_k : *parameter trend in development period k*

τ_t : *parameter trend in payment period t*

$i = 0,1,...,n-1$

$j = 0,1,...,n-1$

$t = 1,2,...,i+j$

The parameters in the accident year direction determine the level from year to year; often the level shows little change over many years, requiring only a few parameters. The parameters in the development year direction represent the trend from one development year to the next. This trend is often linear (on the log scale) across many of the later development years, often requiring only one parameter to describe the tail of the data. The parameters in the payment year direction describe the trend from payment year to payment year. For further explanation about using PTF models to estimate the outstanding claims liability is given below.

3. Example

In this paper, we use the data from Mack (1994). The data are incurred losses for automatic facultative business in general liability, taken from the Historical Loss Development Study, 1991, published by the Reinsurance Association of America (data values in thousand dollars).

Table 1 Run-off triangle of the incremental payments of the AFG data

Accident	*Development year*									
year	0	1	2	3	4	5	6	7	8	9
1981	5012	3257	2638	898	1734	2642	1828	599	54	172
1982	106	4179	1111	5270	3116	1817	-103	673	535	
1983	3410	5582	4881	2268	2594	3479	649	603		
1984	5655	5900	4211	5500	2159	2658	984			
1985	1092	8473	6271	6333	3786	225				
1986	1513	4932	5257	1233	2917					
1987	557	3463	6926	1368						
1988	1351	5596	6165							
1989	3133	2262								
1990	2063									

There are some step to estimate the outstanding claims liability using PTF model :

1. Preliminary analysis of the claims data
2. Identification models
3. Estimation outstanding claims liability

Preliminary analysis of the claims data has main goal to look inside in the data, as we know PTF model required positive valuees for the incremental data claims. If there are some negative values in the incremental data claims, then some modification is needed. Verral and Li (1993) gives the constant solution to solve negative values in the incremental data. Based on Verral and Li method, table 1 can be written

Table 2 Run-off triangle of the incremental payments of the AFG data after modification by constant solution ($\hat{\tau} = 1575.6559$)

Accident year	Development year 0	1	2	3	4	5	6	7	8	9
1981	6588	4833	4214	2474	3310	4218	3404	2175	1630	1748
1982	1682	5755	2687	6846	4692	3393	1473	2249	2111	
1983	4986	7158	6457	3844	4170	5055	2225	2179		
1984	7231	7476	5787	7076	3735	4234	2560			
1985	2668	10049	7847	7909	5362	1801				
1986	3089	6508	6833	2809	4493					
1987	2133	5039	8502	2944						
1988	2927	7172	7741							
1989	4709	3838								
1990	3639									

Identification models has main goal to built or designed the PTF model that captures trend in the incremental data. After some process (include testing model parameter) there is only one PTF model that can be the optimal model, as we can see in equation (2)

$$y(i,j) = \hat{\alpha} + \hat{\gamma}_1 + (j-1)\hat{\gamma}_2 \quad + \varepsilon(i,j)$$
$$E(y(i,j) \mid i,j) = \hat{\alpha} + \hat{\gamma}_1 + (j-1)\hat{\gamma}_2 \qquad (3)$$

with the values of the parameter

$$\hat{\alpha} = 8.1849$$
$$\hat{\gamma}_1 = 0.6054$$
$$\hat{\gamma}_2 = \text{-}0.1773$$
$$\sigma^2 = 0.1291$$

The last step is how to calculate the estimation of the outstanding claims liability. From Modification by constant solution

$$y(i,j) = \ln(p_{i,j} + \hat{\tau})$$

$$p_{i,j} + \hat{\tau} \sim \text{Lognormal}\left(\alpha_i + \sum_{k=1}^{j}\gamma_k + \sum_{t=1}^{i+j}\iota_t, \sigma^2\right)$$
$$E(p_{i,j} \mid i,j) + \hat{\tau} = \exp\left(\alpha_i + \sum_{k=1}^{j}\gamma_k + \sum_{t=1}^{i+j}\iota_t + \frac{1}{2}\sigma^2\right)$$
$$E(p_{i,j} \mid i,j) = \exp\left(\alpha_i + \sum_{k=1}^{j}\gamma_k + \sum_{t=1}^{i+j}\iota_t + \frac{1}{2}\sigma^2\right) - \hat{\tau}$$

from equation (3)

$$E(p_{i,j} \mid i, j) = \exp\left(\hat{\alpha} + \hat{\gamma}_1 + (j-1)\hat{\gamma}_2 + \frac{1}{2}\hat{\sigma}^2\right) - \hat{\tau} \qquad (4)$$

with parameter values

$$\hat{\alpha} = 8.1849$$
$$\hat{\gamma}_1 = 0.6054$$
$$\hat{\gamma}_2 = -0.1773.$$
$$\sigma^2 = 0.1291$$
$$\hat{\tau} = 1575.6559$$

Using equation (4) we get the estimate of the outstanding claims liability

Table 3 the estimate of the outstanding claims liability

Accident	*Development year*									
year	0	1	2	3	4	5	6	7	8	9
1981										
1982										121.1
1983									450.3	121.1
1984								843.3	450.3	121.1
1985							1312.5	843.3	450.3	121.1
1986						1872.8	1312.5	843.3	450.3	121.1
1987					2541.7	1872.8	1312.5	843.3	450.3	121.1
1988				3340.4	2541.7	1872.8	1312.5	843.3	450.3	121.1
1989			4294.1	3340.4	2541.7	1872.8	1312.5	843.3	450.3	121.1
1990		5432.7	4294.1	3340.4	2541.7	1872.8	1312.5	843.3	450.3	121.1

By adding all cell values at table 3, we get the total estimate of the outstanding claims liability $ 62,042,896 or $ 62 million.

4. Sensitivity Analysis

In this section we explore the sensitivity of the estimate of the outstanding claims liability, given a particular forecasting method or model. The sensitivity of the estimate will give insights into the method or model chosen and may assist in setting the reserve. Given a forecasting model, the estimate of the outstanding claims liability is dependent on the observed values. Whatever model is used to estimate the outstanding claims liability, it is important to address how sensitive the estimate is to changes in the observed data.

The sensitivity of the obtained estimate of the outstanding claims liability is analyzed using a tool called *leverage*. Leverage can be used to evaluate the robustness of the model used to estimate the outstanding claims liability (Tampubolon, 2008).

$$Leverage \equiv \frac{\Delta\ estimate\ of\ the\ outstanding\ claims}{\Delta\ incremental\ payment\ in\ a\ cell\ of\ the\ runoff\ triangle} \qquad (5)$$

Given a forecasting method or model, the estimate of the outstanding claims liability is a function of the values in the cells of a runoff triangle. Assuming that the first partial derivative of this function exists, the leverage defined by (5) can be considered as the relative rate of change of the estimate of the outstanding claims liability at the value in that particular cell of the runoff triangle. Therefore, (5) can be used as a measurement of the sensitivity of the estimate of the outstanding claims liability to small perturbations in

the value of each cell of a runoff triangle. Sensitivity is measured by making small perturbations in the data because calculating the first partial derivative algebraically is not a simple matter for PTF model.
From the definition of a rate of change, a leverage value can be zero, positive or negative. A leverage near zero means that a small perturbation in that particular cell causes (almost) no change in the estimate of the outstanding claims liability. In claims reserving, a forecasting methodology which produces almost zero leverage is not desirable since it means that the estimate is largely independent of the corresponding observed data. A positive leverage, say +*k*, means that the estimate of the outstanding claims liability is *increased* by *k* times the change in the observed value, whereas a negative leverage of -*k*, say, means that there is a *decrease* in the estimate of the outstanding claims liability by the amount of *k* times the change in the observed value.

3. Empirical Result

To illustrate the procedures, the AFG data described in Section 3 is used. As mentioned earlier, using the PTF model, the estimates of the outstanding claims liability for the AFG data is $ 62.042 million,respectively. Given the PTF model, the PTF leverage is calculated as follows. Let us say that there is a $1000 increase in cell (0,0) of the runoff triangle of the incremental payments (an increase of $1000 is small enough since increases of $500 and $1 in the incremental payments also result in the same leverage values). The resulting leverage is 0.236. This means that there is an increase in the estimate of the outstanding claims liability of almost 0.2 times the increase in the first cell. In other words, had the claims paid in accident year 0 and development year 0 been $5 013 000 instead of $5 012 000, then the resulting PTF estimate of the outstanding claims liability will be approximately $236 higher than the original estimate of $ 62.042 million. In another example, for the same accident year 0, let us say that there is an increase of $1000 in the paid claims at the final development year (at the tail). Then the change in estimated total outstanding is approximately 5 times as much. This means that, had the $1000 claims been paid later, the resulting PTF estimate of the outstanding claims liability will be approximately $5000 higher than the original PTF estimate. So, for the same accident year 0, if the PTF model is used to forecast future values, a lower estimate of the outstanding claims liability will be obtained if the claims were paid earlier and a higher estimate is obtained if they were paid later. The complete leverage values for all of the cells of the runoff triangle of the AFG data are shown in the runoff triangle of the PTF leverage given by table 4. Each cell of the runoff triangle below shows the leverage (the rate of change) of the estimate of the outstanding claims liability at that particular cell.

Tabel 4.1 PTF leverage

Accident	*Development year*									
year	0	1	2	3	4	5	6	7	8	9
1981	0.236	-0.125	0.128	0.2709	0.879	1.191	1.753	2.904	4.339	5.003
1982	-1.151	-0.028	-0.228	0.478	0.81	1.316	2.597	2.847	3.664	
1983	0.169	0.056	0.252	0.4674	0.839	1.085	2.194	2.901		
1984	0.248	0.068	0.233	0.4744	0.861	1.188	2.047			
1985	-0.283	0.126	0.271	0.4604	0.772	1.58				
1986	-0.124	0.024	0.26	0.3541	0.821					
1987	-0.623	-0.099	0.274	0.3787						
1988	-0.177	0.056	0.27							
1989	0.148	-0.311								
1990	0.01									

Examining table 4, several interesting results may be noted:
1. For accident years 0 up to 6, the PTF leverages of the estimates of the outstanding claims liability are negative for early development years and positive in later developments. In other words, had the $1000 been paid earlier, the PTF estimate of the outstanding claims will be lower than the original estimate and had it paid later due to some delay, the estimate will be higher. Hence, if the PTF model used to forecast future values, delayed (or delaying) payments will result in an increase in the estimate of the outstanding claims liability.

2. The PTF leverage values are positive and reasonably high in the tails and for the data in the last accident year, compared to those at the other cells of the runoff triangle. A higher leverage value means that the estimate of the outstanding claims liability is more sensitive to perturbations.
3. For accident year 0 to 6, going across development years, there is *a zone of (or close to) zero leverage*, at which the PTF method is insensitive to change in the data.
A graphical display of the PTF leverage values of the AFG data is shown in Figure 1. The yellow colour represents zero leverage.

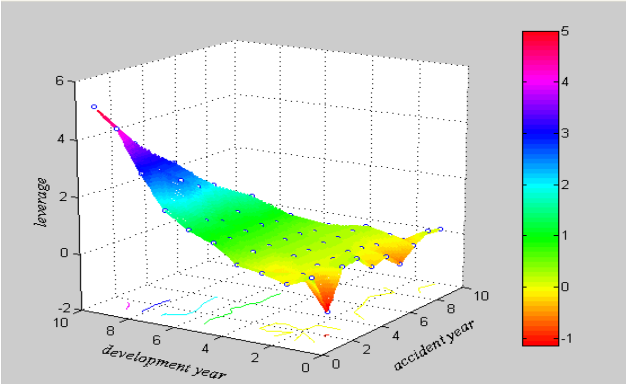

Figure 1. Graphical display of the PTF leverage

References

1. Barnett, G., and Zehnwirth, B. (2000), "Best Estimates for Reserves," *Proceedings of the Casualty Actuarial Society*, **87**, 245-321.
2. Mack, Thomas, "Distribution-Free Calculation of the Standard Error of Chain Ladder Reserve Estimates," *ASTIN Bulletin*, **23**, 2, 1993, pp. 213–225.
3. Mack, Thomas, "Which StochasticModel is Underlying the Chain Ladder Method?" *Insurance Mathematics and Economics*, **15**, 2/3, 1994, pp. 133–138.
4. Reinsurance Association of America, *Historical Loss Development Study, 1991 Edition*, Washington, D.C.: R.A.A., 1991.
5. Tampubolon ,Dumaria R, (2008) " Uncertainties in the Estimation of Outstanding Claims Liability in General Insurance", Ph.D Thesis Macquarie University, Sidney Australia.
6. Verral,Rj., Li Z,1993, "Negatif incremental claims:chain ladder and linear model", *Journal of the Institut of Actuaries* **120**, 171-183