ISSN 1693 - 6698

Volume 16 November 2012 Nomor 2

Makara

SERI TEKNOLOGI

Penerbit:
Direktorat Riset dan Pengabdian Masyarakat
Universitas Indonesia
Kampus Universitas Indonesia
Depok 16424
Indonesia

Kantor:
Gedung DRPM UI, Kampus Universitas Indonesia,
Depok 16424, Indonesia
Telp.+62 21 7270152; 78849118 Fax.+62 21 78849119
Homepage: http://journal.ui.ac.id/technology
E-mail: editor_mst@ui.ac.id

MAKARA Seri TEKNOLOGI merupakan jurnal ilmiah yang menyajikan artikel orisinal tentang pengetahuan dan informasi riset atau aplikasi riset dan pengembangan terkini dalam bidang teknologi. Jurnal ini merupakan sarana publikasi dan ajang berbagi karya riset dan pengembangannya di bidang teknologi. Pemuatan artikel di jurnal ini dialamatkan ke kantor editor, dapat melalui e-mail/journal online/Web. Informasi lengkap untuk pemuatan artikel dan petunjuk penulisan artikel tersedia di dalam setiap terbitan. Artikel yang masuk akan diperiksa oleh mitra bestari dan para editor. Jurnal ini terbit secara berkala sebanyak dua kali dalam setahun (April dan November). Pemuatan naskah tidak dipungut biaya. MAKARA Seri TEKNOLOGI adalah peningkatan dari MAKARA Seri B: Bidang Sains dan Teknologi sebagai penyempurnaan dari Jurnal Penelitian Universitas Indonesia MAKARA yang terbit sejak Januari 1997. Jurnal MAKARA Seri TEKNOLOGI terakreditasi B menurut Keputusan Direktur Jendral Pendidikan Tinggi Kementerian Pendidikan dan Kebudayaan Nomor: 56/DIKTI/Kep/2012 (masa berlaku Juli 2012 s.d. Juli 2017).

MAKARA Seri TEKNOLOGI (MAKARA of Technology Series) is a scientific journal publishing original articles on new knowledge and research or research application with current issues in technology. The journal is published by the Directorate of Research and Community Services, Universitas Indonesia and provides a broad-based forum for the publication and sharing of ongoing research and development in technology. The paper to be presented in this journal is addressed to the editorial office or e-mail. The complete information regarding the procedures to send an article is available in each volume and on its website. All articles will be subjected to double-blind peer review process following a review by the editors. MAKARA Seri Teknologi is an elaboration of MAKARA Seri B: Sains dan Teknologi that was the improvement of Jurnal Penelitian Universitas Indonesia MAKARA, which has been published since Januari 1997. Starting from 2008, the journal has been periodically published twice a year (April and November). Full text articles are available from Volume 6 No. 1 April 2002 free of charge.

ISSN 1693-6698

Makara

SERI TEKNOLOGI

Volume 16 November 2012 Nomor 2

ISSN 1693-6698

Makara

SERI TEKNOLOGI

Volume 16 November 2012 Nomor 2

WORST CASE RELATIVE DISTURBANCE GAIN ARRAY FOR UNCERTAIN DISTILLATION SYSTEM

Rudy Agustriyanto*) and Jie Zhang

1. Chemical Engineering, Surabaya University, Surabaya 60292, Indonesia
2. School of Chemical Engineering and Advanced Materials, Newcastle University, Newcastle upon Tyne, NE1 7RU, UK

*)*E-mail: us6193@yahoo.com*

Abstract

Constrained nonlinear optimization formulation for calculating the worst case lower and upper bounds of relative disturbance gain array (RDGA) for uncertain process models is presented. The proposed approach seeks the minimum and maximum values of the relative disturbance gains subject to the constraints that the process gains and disturbance gains are within their uncertainty ranges. RDGA ranges are useful for control structure determination and the related robustness as they provide information regarding the sensitivity to gain uncertainties. The proposed method is demonstrated by ternary distillation column case study. Closed loop simulation results support the analysis based on the proposed method. It is shown that for a particular degree of uncertainties, the range of process gain determinant should not include zero to ensure the successfulness of the calculation. For the distillation system being studied, the maximum allowable α is 0.339 to avoid the singularity of matrix *K*.

Abstrak

Kondisi Terburuk Harga *Relative Disturbance Gain Array* untuk Sistem Distilasi Tak Pasti. Artikel ini mempresentasikan formulasi optimisasi nonlinear berbatas untuk menghitung kondisi terburuk batas bawah dan batas atas harga *relative disturbance gain array* (RDGA) untuk suatu model proses yang mengandung ketidakpastian. Pendekatan yang diusulkan adalah untuk mencari harga *relative disturbance gain* minimum dan maksimum sesuai batasan kisaran ketidakpastian yang terdapat baik pada *gain* proses maupun *gain* gangguan. Kisaran RDGA berguna untuk penentuan struktur pengendali dan ketegarannya (*robustness*) karena menyediakan informasi terkait sensitivitasnya terhadap ketidakpastian harga *gain*. Metode yang diusulkan kemudian diaplikasikan pada studi kasus kolom distilasi. Hasil simulasi lintas tertutup mendukung analisis yang didasarkan pada metode yang diusulkan. Pada kasus yang dipelajari, ditunjukkan bahwa untuk suatu derajat ketidakpastian tertentu, kisaran determinan *gain* tidak boleh mencakup titik nol untuk menjamin keberhasilan perhitungan. Untuk kasus sistem distilasi yang dipelajari, harga maksimum ketidakpastian, α adalah 0.339 untuk menghindari singularitas matrix *K* (*gain*).

Keywords: distillation control, relative disturbance gain array, relative gain array

1. Introduction

With a given set of controlled and manipulated variables, controllability analysis can then be performed to the system for selecting control configuration [1]. A system is said to be controllable if the controlled variables can be maintained at their setpoints in steady states, in spite of disturbances entering the systems. For a square system, a system is controllable if the determinant of the gain matrix is non zero.

Decentralized (multi-loop) control relies heavily on steady state tools such as the relative gain array (RGA) [2], Niederlinski Index (NI) [3], relative disturbance gain (RDG) and relative normalized gain array (RNGA) [4-5]. RGA has found widespread acceptance both in academia and industry since its introduction over 40 years ago, particularly after the improvement on closed loop stability considerations by using NI as a stability criteria. The RGA–NI rule for decentralized control are summarized as follows [6]: a) The original RGA offers an interaction rule by its size (the paired RGA elements should be the closest to 1 and large RGA elements should be avoided), b) The NI provides a necessary stability condition by its sign (avoid pairings with negative NI), c) The signs of the RGA elements lead to the integrity rules (all the paired RGA elements must be positive), d) The sensitivity of the RGA elements to gain uncertainties presents the robustness rule.

The popularity of RGA is mainly because of its simplicity and confirmed reliability in many case studies. However, RGA has been known to have some deficiencies as it does not consider dynamics and disturbances. Based on the process and disturbance transfer function models, Stanley *et al.* [4] proposed RDG for analyzing the disturbance rejection capability in multi-loop control. RDG overcomes one of the limitations of RGA by allowing disturbances to be included in operability analysis. Chang and Yu extended this concept by introducing relative disturbance gain array (RDGA) and generalized relative disturbance gain array (GRDG) [7].

Recently Chen and Seborg [8] presented an analytical expression for RGA uncertainty bounds. Two types of model uncertainty were considered: worst case bounds, where all elements of the steady state process gain matrix are allowed to change simultaneously within their bounds, and statistical uncertainty bounds. A different method by using the structured singular value (μ) analysis framework was introduced for the calculation of the magnitude of the worst-case relative gain [9].

Agustriyanto and Zhang [10] reported method for calculating uncertainty bounds for relative disturbance gain via optimization for the calculation of RDGA range under model uncertainties. The model uncertainty type considered is worst case bounds. The lower and upper bounds of an RDGA element are calculated as two constrained optimization problems. The method seeks the minimum (for the lower bound) or maximum (for the upper bound) of an RDGA element subject to the constraints that allowable model parameters are within their uncertainty bounds. RDGA ranges are shown to be important for control pairing analysis. In this paper, closed loop simulation were then performed to evaluate the RDGA analysis.

2. Methods

The RDGA matrix of a non singular square matrix K and a vector disturbance Kd can be determined as the following [7]:

$$RDGA=\left[K^{-1}diag(K_d)\right]^{-1}\left[diag\left(K^{-1}K_d\right)\right] \tag{1}$$

where *diag(.)* transforms a vector (.) into a diagonal matrix with each element put on the corresponding diagonal position, that is, the *i*th element of a vector (.) is put on the *ii*th entry of a matrix.

Each element of RDGA matrix is related to the corresponding element of RGA matrix through the following relationship:

$$\beta_{ij}=\lambda_{ij}+\sum_{\substack{k=1\\k\neq i}}^{n}\frac{K_{ij}\breve{K}_{jk}K_{dk}}{K_{di}} \tag{2}$$

where β_{ij} = element of RDGA matrix

λ_{ij} is the ij th element of the RGA

$\breve{K}_{ik}$ is the element on the *i*th row and *k*th column of K^{-1}

The following equation is the relation between *ij*th element of RGA and steady state gains matrix [11]:

$$\lambda_{ij}=(-1)^{i+j}\frac{K_{ij}\det\left(K^{ij}\right)}{\det(K)} \tag{3}$$

In Eq. (3), K^{ij} is the submatrix that remains after the *i*th row and *j*th column of *K* are deleted.

It is obvious that β_{ij} is a function of *K* and K_d, that is

$$\beta_{ij}=f(K,K_d) \tag{4}$$

Assume that the uncertainty bounds for all steady state process and disturbance gains K_{ij} and K_{di} are given, then there will be *2(n2+n)* constraints for all those gains which can be formulated as follows:

$$AX\le b \tag{5}$$

where X is a vector of size $(n2+n)\times 1$ containing all elements of K and K_d as its elements: $X=[K_{11}\ \dots\ K_{nn}\ \ K_{d1}\ \dots\ K_{dn}]^T$, b is a vector of size $(2(n2+n))\times 1$ containing the lower and upper bound values of the corresponding elements of *X*, and *A* is an appropriate matrix of size $(2(n2+n))\times(n2+n)$ satisfying the inequalities in Eq.(5).

Therefore, the lower bound and the upper bound of β_{ij} can be formulated as the following respectively:

Lower bound:

$$\min_X \beta_{ij}=f(X) \tag{6}$$

Upper bound:

$$\max_X \beta_{ij}=f(X) \tag{7}$$

both subject to the constraints in Eq.(4).

Note that β_{ij} cannot be determined if the value of *det(K) = 0*. Furthermore, when *det(K) = 0*, the process will be uncontrollable in that some controlled variables will be dependent to each other and will not be able to follow independent setpoint changes. Therefore, in order to use the above method and also to ensure the process is controllable, the range of *det(K)* should not include 0. The range of *det(K)* can be considered as a function of

all the individual elements of the gain matrix:
$det(K) = g(X)$ (8)

The range of $det(K)$ can be calculated by using the same optimization method:

Lower bound:

$$\min_X \det(K) = g(X) \quad (9)$$

Upper bound:

$$\max_X \det(K) = g(X) \quad (10)$$

both subject to the same constraints in Eq.(5).

The RDGA matrix in conjunction with the structure selection matrix is used to determine the so called GRDG which is useful for control structure selection. A structure selection matrix is an $n \times n$ matrix where the ijth entry is set equal to 1 if the element is chosen for controller pairing or equal to zero if the element is ignored. The value of GRDG vector element is simply the row wise summation of RDGA with the corresponding structure selection matrix.

There are various numerical methods that can be used to solve this constrained optimization problem, such as grid search, random jumping method, the generalized reduced gradient algorithm, etc [12-13]. By using grid search optimization, RDGA in Eq.(1) can be evaluated at all combination points that are specified between the uncertainty bounds of K and K_d in nested loop and hence RDGA ranges are determined by sorting out the minimum and maximum values of each element from all the computed RDGAs. This method requires huge number of RDGA calculation which cannot be avoided. By dividing each element of each gain into only 2 equal segments (3 nodes) then for 3×3 size of K matrix and 3×1 K_d matrix, it will require $3^{(9+3)}$ = 531,441 calculations. This method generally is not preferred since the number of segments/nodes must be increased for more precise calculation. Moreover, most plant wide control problem involves many control, manipulating and disturbance variables which contribute to the rapid increase of the number of calculation.

In random jumping method, random values of each element of K and K_d between their bounds are generated and the RDGA is evaluated at this point. Calculation is performed for large number of random points of K and K_d and the RDGA ranges are picked from the total results. This method is simple and fairly acceptable for this purpose. Other advance optimization techniques may require gradient of the function for generating new point for iteration. However, the availability of the optimizer such as Matlab Optimization Toolbox (e.g. *fmincon*) makes computation become faster and able to provide accurate results without bothering about derivatives of the function which is often difficult to obtain. The formulated problem can be readily solved in this Matlab environment. A satisfactory result can be obtained by initiating the optimization from different starting point within the bounds if the objective function exhibits many local optima.

3. Results and Discussion

In this example we consider the two distillation column system for separating benzene, toluene and m-xylene [14]. The process transfer function matrix, $G(s)$, and the disturbance transfer function matrix, $G_d(s)$, of the Ding and Luyben (DL) column are as follows:

$$G(s) = \begin{bmatrix} \frac{-11.5e^{-s}}{(23s+1)(5s+1)} & 0 & 0 \\ \frac{3.75e^{-2s}}{(14s+1)(3s+1)^2} & \frac{1.6e^{-1.3s}}{(13s+1)(3s+1)} & \frac{-1.2e^{-10.5s}}{(15.5s+1)(3s+1)} \\ \frac{20.6e^{-1.9s}}{(23s+1)(18s+1)} & \frac{-7.5e^{-2.3s}}{(37.3s+1)(2s+1)} & \frac{23.1e^{-s}}{(42s+1)(2s+1)} \end{bmatrix} \quad (11)$$

$$G_d(s) = \begin{bmatrix} \frac{-1.95e^{-5s}}{(12s+1)^2} \\ \frac{1.52e^{-6s}}{(12s+1)^2(5s+1)} \\ \frac{-4.45e^{-7s}}{(40s+1)(10s+1)^2} \end{bmatrix} \quad (12)$$

The process outputs are:
y_1 = composition of benzene from top of column 1
y_2 = composition of toluene from top of column 2
y_3 = composition of m-xylene from bottom of column 2.

The manipulated variables are:
u_1 = heat transfer to reboiler 1
u_2 = reflux rate at column 2
u_3 = heat transfer to reboiler 2

The disturbance variable is:
d = feed composition type 3 (30%, 40%, 30%) or 4 (20%, 60%, 20%).

The nominal value of RDGA calculated using Eq.(1) is:

$$RDGA = \begin{bmatrix} 1 & 0 & 0 \\ 0.42 & 0.41 & 0.17 \\ -0.79 & 0.66 & 1.13 \end{bmatrix} \quad (13)$$

Assume now that the uncertainty bounds for all process and disturbance steady state gain K_{ij} and K_{di} are given by

$$\left|\Delta K_{ij}\right| \le \alpha \left|\hat{K}_{ij}\right| \quad (14)$$

$$\left|\Delta K_{di}\right| \le \alpha \left|\hat{K}_{di}\right| \quad (15)$$

Similar to what have been done for the nominal case [7], GRDG analysis is performed for the three cases of uncertain models. The results are compared to the nominal value analysis. The controller structures are limited to be of diagonal, block diagonal (bd), and full multivariable structures.

The uncertainty ranges for RDGA calculated by random jumping method are shown below in Eq.(16–18) for the case of α =0.01, 0.1 and 0.25 respectively.

$$RDGA_1 = \begin{bmatrix} 1 \le \beta_{11} \le 1 & 0 \le \beta_{12} \le 0 & 0 \le \beta_{13} \le 0 \\ 0.40 \le \beta_{21} \le 0.43 & 0.38 \le \beta_{22} \le 0.44 & 0.16 \le \beta_{23} \le 0.19 \\ -0.81 \le \beta_{31} \le -0.76 & 0.60 \le \beta_{32} \le 0.72 & 1.05 \le \beta_{33} \le 1.21 \end{bmatrix} \quad (16)$$

$$RDGA_2 = \begin{bmatrix} 1 \le \beta_{11} \le 1 & 0 \le \beta_{12} \le 0 & 0 \le \beta_{13} \le 0 \\ 0.28 \le \beta_{21} \le 0.62 & 0.06 \le \beta_{22} \le 0.68 & 0.02 \le \beta_{23} \le 0.37 \\ -1.15 \le \beta_{31} \le -0.54 & 0.08 \le \beta_{32} \le 1.52 & 0.19 \le \beta_{33} \le 2.01 \end{bmatrix} \quad (17)$$

$$RDGA_3 = \begin{bmatrix} 1 \le \beta_{11} \le 1 & 0 \le \beta_{12} \le 0 & 0 \le \beta_{13} \le 0 \\ 0.16 \le \beta_{21} \le 1.10 & -1.16 \le \beta_{22} \le 1.07 & -0.32 \le \beta_{23} \le 1.32 \\ -1.95 \le \beta_{31} \le -0.30 & -1.71 \le \beta_{32} \le 4.02 & -2.23 \le \beta_{33} \le 4.11 \end{bmatrix} \quad (18)$$

As a comparison, Eq. (19-21) below are the results computed by using Matlab Optimization Toolbox. It is shown that wider ranges can be obtained compared to previous results using random jumping method. Therefore the GRDG analysis for uncertain system presented in this paper will be based on the RDGA results computed by Matlab Optimization Toolbox due to its accuracy

$$RDGA_1 = \begin{bmatrix} 1 \le \beta_{11} \le 1 & 0 \le \beta_{12} \le 0 & 0 \le \beta_{13} \le 0 \\ 0.40 \le \beta_{21} \le 0.44 & 0.37 \le \beta_{22} \le 0.45 & 0.15 \le \beta_{23} \le 0.20 \\ -0.82 \le \beta_{31} \le -0.75 & 0.57 \le \beta_{32} \le 0.75 & 1.02 \le \beta_{33} \le 1.23 \end{bmatrix} \quad (19)$$

$$RDGA_2 = \begin{bmatrix} 1 \le \beta_{11} \le 1 & 0 \le \beta_{12} \le 0 & 0 \le \beta_{13} \le 0 \\ 0.28 \le \beta_{21} \le 0.62 & -0.11 \le \beta_{22} \le 0.74 & -0.02 \le \beta_{23} \le 0.49 \\ -1.17 \le \beta_{31} \le -0.53 & -0.18 \le \beta_{32} \le 1.77 & -0.13 \le \beta_{33} \le 2.26 \end{bmatrix} \quad (20)$$

$$RDGA_3 = \begin{bmatrix} 1 \le \beta_{11} \le 1 & 0 \le \beta_{12} \le 0 & 0 \le \beta_{13} \le 0 \\ 0.15 \le \beta_{21} \le 1.16 & -3.51 \le \beta_{22} \le 1.93 & -1.08 \le \beta_{23} \le 3.35 \\ -2.18 \le \beta_{31} \le -0.28 & -7.98 \le \beta_{32} \le 8.60 & -7.13 \le \beta_{33} \le 11.16 \end{bmatrix} \quad (21)$$

GRDG analysis for the nominal model (Table 1) shows that the block diagonal controller bd [(1,3),2] offers the best disturbance rejection capability [7]. Small values of GRDG elements are preferable since they reflect the ratio of net load effect over the open loop load effect. The GRDG values for the three cases of model uncertainties are presented in Table 2.

For α = 0.01, it is obvious from Table 2 that the block diagonal controller bd [(1,3),2] will be recommended. However, as the value of α increased to 0.1 and 0.25, it can be predicted that bd [(1,3),2] will be no longer the best choice. The performance of this control structure may not be as good as the diagonal control structure.

Table 1. GRDG for the Nominal Model of the DL Column

Control Structure	GRDG		
Diagonal	[1.00	0.41	$1.13]^T$
bd [(1,2),3]	[1.00	0.83	$1.13]^T$
bd [(1,3),2]	[1.00	0.41	$0.34]^T$
bd [(2,3),1]	[1.00	0.58	$1.79]^T$
Full	[1.00	1.00	$1.00]^T$

Table 2. GRDG for Uncertainty Models of the DL Column

Control structure	$\alpha = 0.01$	$\alpha = 0.1$	$\alpha = 0.25$
Diagonal	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ 0.3695 \le \delta_2 \le 0.4488 \\ 1.0215 \le \delta_3 \le 1.2324 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ -0.1101 \le \delta_2 \le 0.7390 \\ -0.1254 \le \delta_3 \le 2.2611 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ -3.5142 \le \delta_2 \le 1.9336 \\ -7.1287 \le \delta_3 \le 11.1648 \end{bmatrix}$
bd [(1,2),3]	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ 0.7714 \le \delta_2 \le 0.8842 \\ 1.0215 \le \delta_3 \le 1.2324 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ 0.1699 \le \delta_2 \le 1.3639 \\ -0.1254 \le \delta_3 \le 2.2611 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ -3.3636 \le \delta_2 \le 3.0956 \\ -7.1287 \le \delta_3 \le 11.1648 \end{bmatrix}$
bd [(1,3),2]	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ 0.3695 \le \delta_2 \le 0.4488 \\ 0.2045 \le \delta_3 \le 0.4782 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ -0.1101 \le \delta_2 \le 0.7390 \\ -1.2980 \le \delta_3 \le 1.7356 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ -3.5142 \le \delta_2 \le 1.9336 \\ -9.3091 \le \delta_3 \le 10.8822 \end{bmatrix}$
bd [(2,3),1]	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ 0.5188 \le \delta_2 \le 0.6439 \\ 1.5899 \le \delta_3 \le 1.9803 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ -0.1292 \le \delta_2 \le 1.2242 \\ -0.3017 \le \delta_3 \le 4.0287 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ -4.5984 \le \delta_2 \le 5.2858 \\ -15.1130 \le \delta_3 \le 19.7644 \end{bmatrix}$
Full	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ 0.9207 \le \delta_2 \le 1.0793 \\ 0.7729 \le \delta_3 \le 1.2261 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ 0.1508 \le \delta_2 \le 1.8491 \\ -1.4743 \le \delta_3 \le 3.5032 \end{bmatrix}$	$\begin{bmatrix} 1 \le \delta_1 \le 1 \\ -4.4478 \le \delta_2 \le 6.4478 \\ -17.2934 \le \delta_3 \le 19.4818 \end{bmatrix}$

Figure 1 shows that the maximum allowable α for this system is 0.339 to avoid singularity of matrix *K*. Closed loop simulation was then performed to evaluate the control performance. PI controllers were used. For tuning purpose, the system is disturbed by the sequence of step disturbance as shown in Figure 2 and the following set-point changes: y_1 set-point was changed from 0 to 1 at t = 450 min; y_2 set-point was changed from 0 to -1 at t = 550 min; y_3 set-point was changed from 0 to 1 at t =650 min.

The output values are recorded for 1000 min simulation time with 1 min sampling time. Table 3 shows the controller parameters which were obtained via optimization for both disturbance rejection and set point tracking during the specified time by minimizing the sum squared of error (SSE).

Simulations were then performed for the arbitrarily altered process and disturbance gains (which reflect model uncertainties) as follows:

$$\overline{G} = \begin{bmatrix} G_{11}(1-\alpha) & G_{12}(1+\alpha) & G_{13}(1-\alpha) \\ G_{21}(1+\alpha) & G_{22}(1-\alpha) & G_{23}(1+\alpha) \\ G_{31}(1-\alpha) & G_{32}(1+\alpha) & G_{33}(1-\alpha) \end{bmatrix} \quad (22)$$

$$\overline{G_d} = \begin{bmatrix} G_{d1}(1-\alpha) \\ G_{d2}(1+\alpha) \\ G_{d3}(1-\alpha) \end{bmatrix} \quad (23)$$

The profile of the considered disturbance in simulation is similar to that used for tuning purpose (Figure 2) but with larger magnitude: Δ*d* = +5 at *t* = 50 min; Δ*d* = +5 at *t* = 150 min; Δ*d* = -15 at *t* = 250 min; Δ*d* = +5 at *t* = 350 min.

Figure 3 shows the results for three different values of *α* ranging from 0 to 0.25. It is shown that closed loop performance deteriorates as the level of model uncertainties is increased. For *α* = 0.339 as shown in Figure 4, the control structure bd[(1,3),2] with controller parameters obtained based on the nominal model obviously fails to achieve the required control objectives.

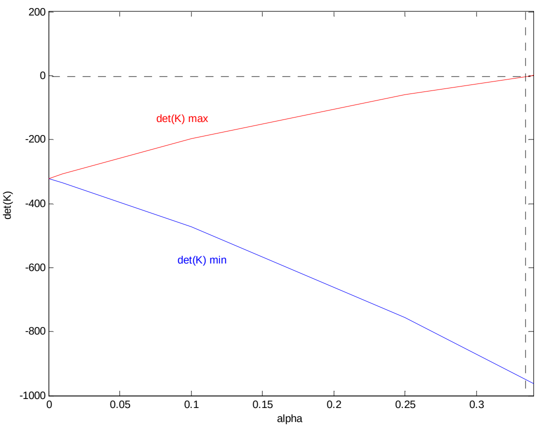

Figure 1. Range of det(*K*) vs *α*

GRDG analysis for uncertain models indicates that for increased values of *α* (0.1 and 0.25) control structure bd[(1,3),2)] may not be the best choice compared to the diagonal control structure. In order to verify the above analysis, closed loop performance was also investigated for the diagonal control structure. Table 4 shows the controller parameters which were obtained via optimization for the diagonal control structure (y_1-u_1, $y_2 - u_2$, $y_3 - u_3$).

Table 3. Controller Parameters for the Control Structure bd[(1,3),2]

Controller	K_c	T_i
$G_{c,11}$	-1.14	36.55
$G_{c,31}$	0.01	0.29
$G_{c,13}$	-0.09	2.55
$G_{c,33}$	1.19	15.42
$G_{c,22}$	1.18	9.49

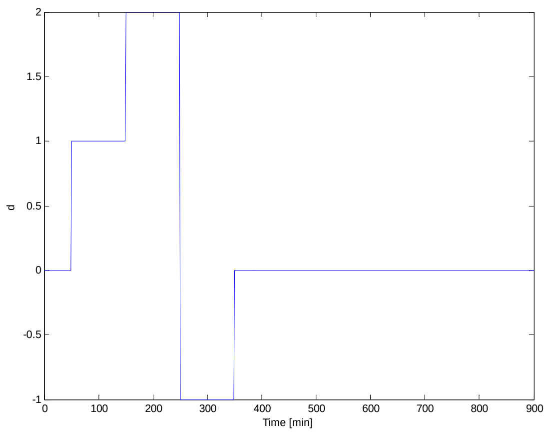

Figure 2. Profile of Disturbance for Controller Tuning

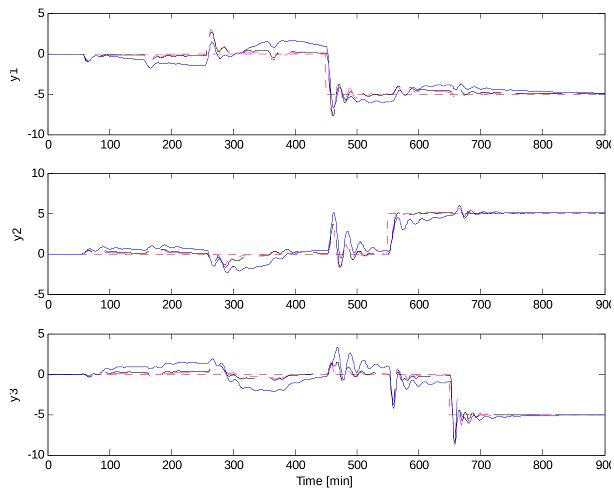

Figure 3. Simulation Results for Example 2, bd[(1,3),2], (*y* setpoint, -·-·-·-·*y* for *α* = 0, - - - - - *y* for *α* = 0.1, ——— *y* for *α* = 0.25)

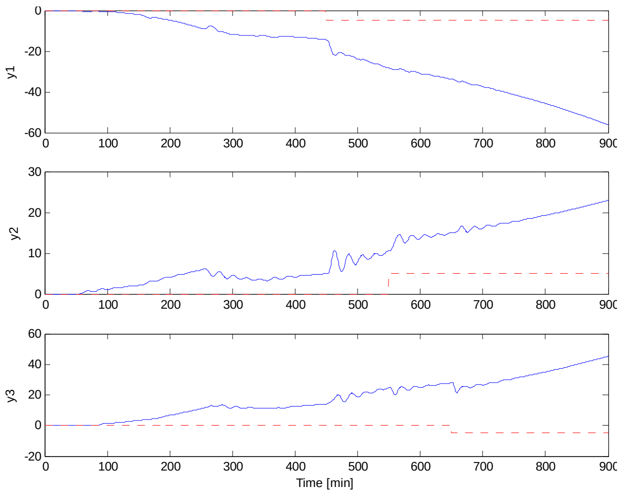

Figure 4. Simulation Results for Example 2, bd[(1,3),2], α = 0.339, (·······y setpoint, —— y for α = 0.339)

Table 4. Controller Parameters for the Diagonal Control Structure

Controller	K_c	T_i
$G_{c,11}$	-1.15	33.32
$G_{c,22}$	1.08	8.53
$G_{c,33}$	1.17	13.29

Figure 5 shows the simulation results for three different values of α (α = 0, 0.1 and 0.25). It can be seen that the diagonal control structure with nominal controller settings gives better performance than bd[(1,3),2] when α = 0.25. The SSE values from the two different control structures are presented in Table 5. It is shown that for small values of α (α = 0 and 0.01), the control performance of bd[(1,3),2] is better than that of the diagonal structure, i.e. with lower SSE values. The results support the above GRDG analysis that as the value of α is increased to 0.1 or 0.25 (Table 4), it becomes harder to see that bd[(1,3),2] is the best choice or not.

The closed loop performance in term of SSE as presented in Table 5 were obtained based on controller setting for the nominal process (i.e. α = 0). However, for uncertain systems, it is not necessary to find controller settings for the nominal model. Table 6 provides alternative controller settings for both bd[(1,3),2] and diagonal structures which were obtained based on the altered gains in Eq. (22) and Eq. (23) for different values of α. Optimization tuning method was used to obtained the best performance (minimum SSE) for each case. Each set of controller setting was then tested on other values of α and their closed loop performances in term of SSE are compared in Table 7. The following conditions are used for both tuning and simulation purposes: 1) A series of step disturbance (Δd) = +1, +1, -3 and +1 at t = 50, 150, 250 and 350 min respectively, 2) y_1 set-point was changed from 0 to 1 at t = 450 min, 3) y_2 set-point was changed from 0 to -1 at t = 550 min, 4) y_3 set-point was changed from 0 to 1 at t = 650 min, 5) simulation time = 1000 min.

Closed loop performance comparison presented in Table 7 can be summarized as the following: 1) For controller parameters which were obtained based on the altered gains at α = 0.01, similar closed loop performance as that using nominal model based settings are obtained. On these settings, bd[(1,3),2] gives better performance (smaller SSE values) when tested on low α values (0, 0.01 and 0.1) while the diagonal control structure gives better performance on other α values, 2) Controller parameters based on α = 0.1 also show that bd[(1,3),2] is better when the system is tested on α =0, 0.01 and 0.1 for the specified altered process, 3) By using controller parameters which were obtained based on α = 0.25, the closed loop performance shows that the diagonal control structure is better for all cases, 4) Both bd[(1,3),2] and diagonal control structures give relatively similar SSE values when tested on α = 0.1. This evidence support the GRDG prediction that as the value of α increased to 0.1 and 0.25, it becomes harder to see that bd[(1,3),2] is the best choice or not.

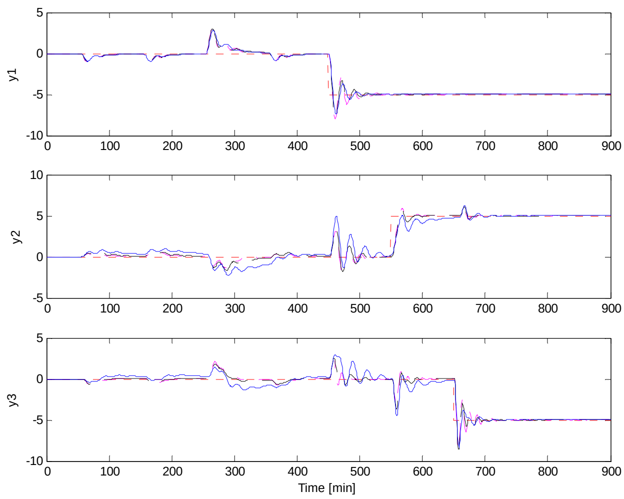

Figure 5. Simulation Results for Example 2, Diagonal, α = 0, 0.1 and 0.25, (·····y setpoint, -·-·-·-y for α = 0, y for α = 0.1, —— y for α = 0.25)

Table 5. SSE Comparison between bd[(1,3),2)] and the Diagonal Control Structure

α	bd[(1,3),2]	Diagonal
0	835.75	937.80
0.01	842.69	939.52
0.1	977.78	996.70
0.25	2238.6	1601.10
0.339	1.8131×10^{6}	3314.60

Table 6. Controller Parameters based on Different Values of *α*

α	Control Structure	Controller	K_c	T_i
0.01	bd[(1,3),2]	$G_{c,11}$	-1.14	32.01
		$G_{c,31}$	0.10	1.81
		$G_{c,13}$	-0.23	6.24
		$G_{c,33}$	1.26	15.17
		$G_{c,22}$	1.19	8.76
	diagonal	$G_{c,11}$	-1.16	33.65
		$G_{c,22}$	1.07	8.31
		$G_{c,33}$	1.18	13.43
0.1	bd[(1,3),2]	$G_{c,11}$	-1.20	41.09
		$G_{c,31}$	2.9896×10^{-4}	0.01
		$G_{c,13}$	-0.14	4.37
		$G_{c,33}$	1.38	15.49
		$G_{c,22}$	1.11	7.55
	diagonal	$G_{c,11}$	-1.23	37.13
		$G_{c,22}$	0.96	6.62
		$G_{c,33}$	1.30	14.24
0.25	bd[(1,3),2]	$G_{c,11}$	-1.14	33.35
		$G_{c,31}$	0.0011	1
		$G_{c,13}$	-0.19	21
		$G_{c,33}$	1.62	10.57
		$G_{c,22}$	0.82	4.46
	Diagonal	$G_{c,11}$	-1.25	37.42
		$G_{c,22}$	0.73	4.25
		$G_{c,33}$	1.49	10.07

Table 7. SSE Comparison for bd[(1,3),2] and Diagonal Control Structures for Different Controller Settings

α as controller setting basis	α for closed loop performance test	bd[(1,3),2] SSE	diagonal SSE
0.01	0	32.85	37.41
	0.01	33.09	37.46
	0.1	38.05	39.50
	0.25	116.39	60.53
	0.339	12327	134.98
0.1	0	34.81	38.22
	0.01	34.79	38.12
	0.1	36.81	38.94
	0.25	73.24	56.75
	0.339	474.71	126.30
0.25	0	82.04	72.74
	0.01	74.20	66.34
	0.1	45.87	45.04
	0.25	50.90	51.53
	0.339	94.82	94.66

4. Conclusions

An alternative method for determining worst case lower and upper bounds RDGA ranges for uncertain process models is presented in this paper. Constrained optimization is used to find the uncertain RDGA ranges. The proposed method is applied to the ternary distillation column. It is shown that the proposed method is easy to use and gives accurate results. Closed loop simulation results confirm the analysis based on the proposed method.

Acknowledgement

This work was supported by Technological and professional Skills Development Sector Project (TPSDP) –ADB Loan No. 1792 –INO. This support is gratefully acknowledged.

References

[1] S. Skogestad, Comput. Chem. Eng. 28 (2004) 219.
[2] E. Bristol, IEEE Trans. Autom. Control 11/1 (1996) 133.
[3] A. Niederlinski, Automatica 7/6 (1971) 691.
[4] G. Stanley, M. Marino-Galarraga, T.J. McAvoy, Ind. Eng. Chem. Process Des. Dev. 24/4 (1985) 1181.
[5] M.J. He, W.J. Cai, W. Ni, L.H. Xie, J. Process Control 19 (2009) 1036.
[6] Z.X. Zhu, Ind. Eng. Chem. Res.35/11 (1996) 4091.
[7] J.W. Chang, C.C. Yu, AICHE J. 38/4 (1992) 521.
[8] D. Chen, D.E. Seborg, AICHE J. 48/2 (2002) 302.
[9] V. Kariwala, S. Skogestad, J.F. Forbes, Ind. Eng. Chem. Res. 45/5 (2006) 1751.
[10] R. Agustriyanto, J. Zhang, Proc. of the 2007 American Control Conference, New York, USA, 2007, p.5360.
[11] P. Grosdidier, M. Morari, B.R. Holt, Ind. Eng. Chem. Fundam. 24/2 (1985) 221.
[12] S.S. Rao, Engineering Optimization–Theory & Practice, 4th ed., John Wiley & Sons, Inc, 2009, p.840.
[13] T.F. Edgar, D.M. Himmelblau, L.S. Lasdon, Optimization of Chemical Processes, McGraw-Hill, 2001, p.672.
[14] S.S. Ding, W. Luyben, Ind. Eng. Chem. Res. 29/7 (1990) 1240.