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PROCEEDING

THE 3RD INTERNATIONAL CONFERENCE ON MATHEMATICS AND STATISTICS

BOGOR, 5 – 6 AUGUST 2008

Mathematics and Statistics: bridge for academia, business, and government in the entrepreneurial era

3rd ICoMS 2008

Department of Statistics
Department of Mathematics
Institut Pertanian Bogor

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Mathematics and Statistics: bridge for academia, business, and government in the entrepreneurial era

organized by

MSMSSEA (Moslems Statisticians and Mathematicians Society in South East Asia)

Department of Statistics
Department of Mathematics
Institut Pertanian Bogor

Department of Mathematics
Universiti Malaysia Terengganu,
Malaysia

PREFACE

Assalaamu'alaikum warahmatullaahi wabarakaatuh

Welcome all participants of ICoMS 2008 to Bogor – Indonesia. This event is organized by MSMSSEA in collaboration with Institut Pertanian Bogor (Indonesia) and Universiti Malaysia Terenganu (Malaysia).

We, the organizing committee, are very glad having this international conference due to many reasons.

1. ICoMS is a good avenue for mathematicians, statisticians, and other scientist to communicate.
2. ICoMS 2008 has a theme related to entrepreneurial era which is very important for mathematicians and statisticians, and scientist in general.
3. The event is important venue for business group, government, and academia to communicate and share knowledge as well.
4. Bogor is beautiful place in Indonesia surrounded by many research centers, IPB, Botanical garden, an other point of interest related to research institution.

We are also happy that the Vice President of Republic of Indonesia, Ministry of National Education, Ministry of Energy and Mineral Resources, and Ministry of Communication and Information Technology are supporting to the ICoMS 2008.

This event held on two days, August 5-6, and consist of several parts. We invite 17 outstanding professors to share and discuss topics in mathematics and statistics, including application. As many as 170 paper and 30 posters presented during this two-day conference. We appreciate to all of contributor from various countries who are motivated to participate in this event.

High appreciation is also awarded to companies and agencies which facilitate so that the even could run well.

We really hope all participants can benefit many things from this international event. May God bless you.

Wa'alaikumsalam warahmatullaahi wabarakaatuh.

The Committee of ICoMS 2008

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POSTER

IS-LM IN SLOW-FAST SYSTEM

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Abstract. *Our real life problem usually is solved by forming the mathematics model. In economics, we found some mathematics model about business cycle. One of the well-known models is IS-LM model. The model, first developed by Sir John Hicks and Alvin Hansen in linear equation system, has been used from 1937. We developed IS-LM model in ordinary differential equation system and observed the behavior change to the time. The model has been developed by Kalecky and Cai in time-delay form. In this paper, we discuss the model in slow-fast system. By using singular perturbation method is obtained qualitative analysis about the equilibrium point of the system and the economical interpretation.*

Keywords : IS-LM model, slow-fast system, singular perturbation

1. Introduction

In the macroeconomics, we know three kinds of market in object views. They are goods market, money market and labor market. The analysis of good market and money market is well-known as IS-LM analysis [1]. The IS-LM model, first developed by Sir John Hicks and Alvin Hansen, has been used from 1937 onwards to summarize a major part of Keynesian macroeconomics [2]. *I* is investment function, *S* is the saving function, *L* is the demand of money and *M* is money supply. In 2005 Cai develop dynamic model of ISLM business cycle [3]. The development of the model come first from Torre :

$$\begin{aligned} \dot{Y} &= \alpha\left(I(Y,r)-S(Y,r)\right) \\ \dot{r} &= \beta(L(Y,r)-M) \end{aligned},$$

with $Y(t)$ is the groos-product and $r(t)$ is the interest rate.
And then, continuing by Gabish and Lorenz :

$$\begin{aligned} \dot{Y} &= \alpha\left(I(Y,r)-S(Y,r)\right) \\ \dot{r} &= \beta(L(Y,r)-M) \\ \dot{K} &= I(Y,K,r)-\delta K \end{aligned}$$

with $K(t)$ is the capital stock.
Cai analyzes model in linear dynamic and uses time delay based on Kalecki's idea :

$$\begin{aligned} \dot{Y} &= \alpha\left(I(Y,r)-S(Y,r)\right) \\ \dot{K} &= I(Y(t-T,K,r)-\delta K \\ \dot{r} &= \beta(L(Y,r)-M) \end{aligned}$$

Now in this paper, we develop the model in non-linear dynamic and use slow fast system :

$$\begin{aligned} \varsigma\dot{r} &= (L(Y,r)-M) \\ \dot{Y} &= \left(I(Y,r)-S(Y,r)\right) \\ \dot{K} &= \varepsilon\left(I(Y,K,r)-\delta K\right) \end{aligned} \tag{1}$$

We assume that the changes rate of *r*, *Y* and *K* to the time differ. Variables *r*, *Y* and *K* are called fast, intermediate, and slow components of system [4, 5]. Parameters $\varsigma > 0$ and $\varepsilon > 0$ are small parameters and they indicate the level of each component.

The second assumption is the investment function *I*, the saving function *S*, the demand for money *L* and the money supply *M* depend nonlinearly on their arguments, that is

$$
\begin{aligned}
I &= a_1 Y - lK - h_1 r^2 \\
S &= a_2 Y + h_2 r^2 \\
L &= pY - mr^2 \\
M &= M - nr
\end{aligned}
\qquad (2)
$$

with $a_1, a_2, h_1, h_2, l, m, n, p$ positive constants [0,1].

From (1) and (2), we get the model and can start observe the behavior change of r, Y and K to the time. We do qualitative analysis using singular perturbation methods. In this paper we discuss the first step of our research, that is, we analyze the model with K constant. Under certain conditions, we obtain the stability cycle of the system by perturbation methods.

2. Stability analysis

As the explaination in the first section, now the model that we analyze is

$$
\begin{aligned}
\varsigma \dot{r} &= nr - mr^2 + pY - M \\
\dot{Y} &= aY - hr^2 - lK
\end{aligned}
\qquad (3)
$$

with a and h positive constants.

Now, the system (3) can called as slow-fast system [4]. By rescaling time $\bar{t} = \frac{t}{\varsigma}$, it is obtained

$$
\begin{aligned}
\dot{r} &= nr - mr^2 + pY - M \\
\dot{Y} &= \varsigma\left(aY - hr^2 - lK\right)
\end{aligned}
\qquad (4)
$$

Let $\varsigma = 0$, we obtain from (4)

$$
\begin{aligned}
\dot{r} &= nr - mr^2 + pY - M \\
\dot{Y} &= 0
\end{aligned}
$$

The orbit of fast subsystem can be construction from r-nullcline. We get parabolics curve ***r***-nullcline

$$Y = \frac{mr^2 - nr + M}{p}$$

with the stationer point P$\left(\frac{n}{2m}, \frac{n^2 - 4mM}{-4mp}\right)$. Our discuss is in the first quadrant. Thus the parabolic should be above r-axis, that is $M > \frac{n^2}{4m}$. The orbits of fast subsystem move in r direction. For the above region of the parabolic curve, we find that $\dot{r} > 0$ such that the orbits direction is horizontal left. On the other hand, in the below area of the parabolic curve, we find that $\dot{r} < 0$. It means the orbits direction move to the right. We get the orbits of the fast subsystem as shown in figure 2.1.

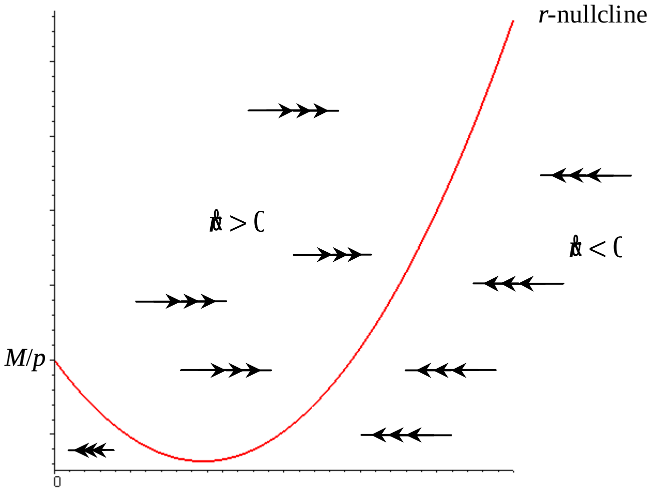

Figure 2.1. The orbits of the fast subsystem

To know the dynamic of gross-product, we take ς =0 in (3), we obtain slow subsystem

$$\begin{aligned} 0 &= nr - mr^2 + pY - M \\ \dot{Y} &= aY - hr^2 - lK \end{aligned} \qquad (5)$$

The orbits of slow subsystem (5) on the surface of r-nullcline $Y=\frac{mr^2-nr+M}{p}$. We find the stable condition if $\frac{lK}{a}<\frac{M}{p}$, and if the cross-section between r-nullcline and Y-nullcline occurs in the minimum point of r-nullcline $P_1\left(\frac{n}{2m},\frac{n^2-4mM}{-4mp}\right)$ and in the point with ordinat value more than M/p, for example, we can take $P_2\left(1,\frac{m-n+M}{p}\right)$ with $h=\left(p(n^2-4m^2)a\right)/\left(m(-n^2-4m^2+4nm)\right)$ and lK = $(-5\,n^2m^2 + n^3\,m - n^2mM + 4\,m^2M^2 - 4\,m^4 + 8m^3n - 4m^3M + 4\,m^2nM- 4m^3M - 4m^2M - n^2p^2 - 4m^2p)a/4(mp)^2$. See figure2.2. We obtain that Y-nullcline is $Y=\frac{hr^2+lK}{a}$, that is a parabolic curve. For region above the curve, we obtain $\dot{Y} > 0$. It means that the orbits directions go down. In the below of the curve, we get $\dot{Y} < 0$. It shows that the orbits direction move up. Thus, we now obtain the slow and fast orbits. See figure 2.2.

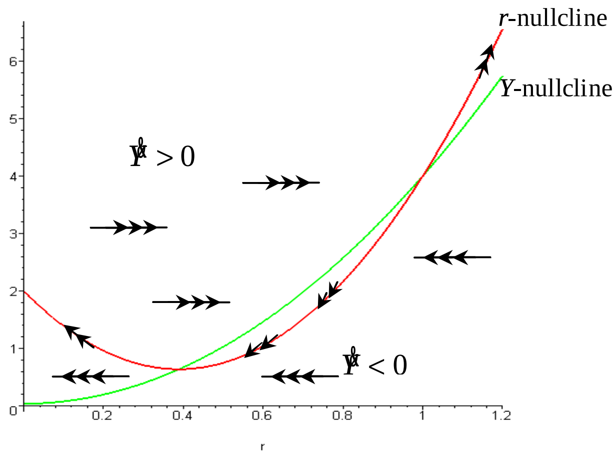

Figure 2.2. Slow-fast orbits

Now, let we take a starting point $A(r_0,Y_0)$ with $r_0 > \frac{n}{2m}$ and Y_0 in the interval of cross-section of *r*-nullcline and *Y*-nullcline curves. We obtain that fast orbits bring A to the stable part of *r*-nullcline, i.e., point B. Slow orbits bring it to point C and then move to point D. When *r* is going to zero, fast orbits bring point D to point E. After that, the orbits will back to C-D-E and continuing in such a way. Thus, we get stable cycle as shown in figure 2.3.

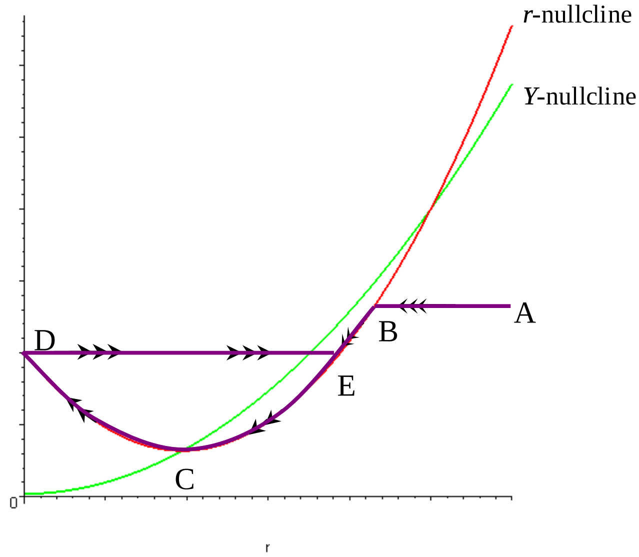

Figure 2.3. stable cycle of the system

If the starting point did not fulfill the condition then we get unstable condition. See figure 2.4.

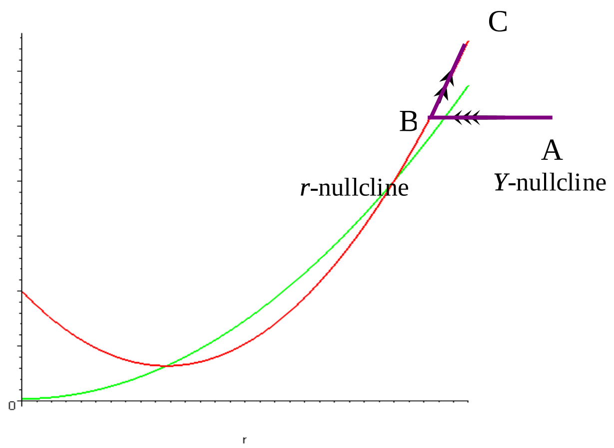

Figure 2.4. unstable condition

3. Discussion

In the second section, we have found the behavior of the system. We obtain stable cycle under some certain condition. In the economical interpretation, it represent of business cycle. Let the starting point interest rate and gross-product r_0 and Y_0 under certain condition as explaination above. In the beginning of cycle the change rate of interest *r* is gradually down until certain point, the value of groos-product *Y* will be change. Follow the slow orbit, the value of interest and groos-product gradually decrease until the minimum turning point of interest null-cline curve. And then, the value of gross-product will be gradually up, at the time, the interest rate is going to zero until a certain point the fast orbit will attract the interst rate such that the value gradually increase and the condition will be repeat.

Real business problem is very complicated. We will always improve the model also the analysis methods such as the model and the analysis result closely to the real problem.

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