

# Implementing Directed Pairwise Judgement Approach in Web-Based AHP Survey Application to Reduce Inconsistency Ratio

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**Abstract.** In the research combining the MCDA method and public participatory approach, an application that collects public opinion is needed. The input from respondents often does not meet the expected standards; example, the consistency ratio of the AHP pairwise judgment. This paper is a part of the primary research with title A GIS-based Multi-Criteria Decision Analysis and Public Participatory Approach for Public Schools Site Selection, and it focused on the AHP pairwise judgment phase. This phase needs to be reconducted because the first pairwise judgment survey application got a poor consistency ratio. The second application implemented a new approach called directed pairwise judgment. This approach has successfully raised the consistency ratio significantly and also reduces the consuming time of the pairwise comparison phase.

**Keywords:** MCDA, Pairwise Judgement, AHP, Consistency Ratio

## INTRODUCTION

MCDA and GIS have been used together as powerful tools to solve various spatial problems in many areas. The AHP technique is a popular method in the MCDA phase. This paper is a part of GIS-based MCDA research in finding optimal school site selection in Surabaya, Indonesia. Figure 1 shows the complete methodology schema for this research, and this paper focuses on the AHP application marked with a red circle.

The MCDA phase begins after all of its factors have been built. It uses a web-based application that collects opinions from invited government officials, school officials, researchers, and experts in urban planning. The MCDA phase got six factors in the form of the raster layers which are: administration factor, population factor, transportation factor, environment factor, student flow factor, and public preference factor [1]. The MCDA phase has been conducted by using a web-based GIS application in 2016, unfortunately, it brought an inadequate result. By using the AHP method with 137 respondents, it just got 5% of the respondent's pairwise judgment with consistency ratio (CR) below 0.1. According to Saaty [2], 90% of respondent pairwise judgment is inconsistent and cannot be accepted. This paper is focused on the process of fixing this unfortunate result.

The two major problems in the pairwise judgment phase are uncomplete pairwise judgment and inconsistent pairwise judgment. In the uncomplete pairwise judgment, researchers develop methods for completing the missing element with a value that leads to higher consistency. It can just follow the rule of consistency and complete the matrix coherently with the available judgments [3]. It also can get the missing element by using experience from a neural network system that already trained with all possible high consistency matrixes [4]. It also can adopt an algorithm that initially used to improve inconsistency, such as linearization method [5]. However, the well-known approach called the connecting path method (CPM) guarantees a minimal geometric consistency index in estimating missing judgment [6]. The uncomplete judgment problem is more straightforward to solve than complete judgment with high inconsistency. Several researchers have done different methods to improve the consistency of pairwise comparison.

In general, those methods are trying to modify as little as possible the problematic matrix element. Those methods include Goal Programming [7], Mathematical model [8], Heuristic method [9], Distance-based inconsistency reduction [10], Particle Swarm optimization [11], Multi-objective Evolutionary computing [12], Linearization [13], Integrated Linear Programming [14], Adapting Hadamard model [15], Abelian Linearly Ordered Group [16], Non-Linear Programming [17], Triad-by-triad inconsistency reduction [18], Geometric framework [19]. All the above method are used after the judges finished their pairwise comparison and is not able to do a review and fix the inconsistent pairwise.

Some other researchers choose to inform the judges about their inconsistent pairwise comparison and then fixing it. Ishizaka and Lusti develop an expert system module that intervening judges right after they finish comparing each pairwise and generating inconsistency. The module explains why inconsistency happens and suggests a consistent alternative [20]. A graphical method called the Gower plot applied to detect ordinal and cardinal inconsistencies. A multiobjective optimization program is proposed to assist the decision-maker in adjusting the preferences [21]. The feedback that informs the consistency ratio can also be given to the judge right after they finish all pairwise comparisons. The judge was suggested with an automatic correction that can be accepted or not. If not accepted, the judge can still also make another adjustment based on the proposed revision [22].

This research will implement inconsistency prevention rather than consistency improvement. The web-based application will provide a direction that leads to consistent pairwise judgment. The application also will give the consistency ratio value as feedback right after the judge completing the pairwise comparison. The second section will explain how the data structure used and how the process worked in the application.

The first web-based application to collect the respondent's judgments [23], which runs about 11 months starting July 2016 with 137 respondents. The respondent came from invited government officials, school officials, researchers, and experts in urban planning. The respondent's judgment was stored in a tabular database. Each data from respondents was stored in a row. Their pairwise selections were stored in different fields in that row. These fields are described in Table 1 below. Each field with initial 'P' (stands for pairwise) in the above table corresponds with a row in the pairwise judgments interface. In each row, 9 options had to be chosen by the respondent. Each option had a different value when stored.

**TABLE 1.** Fields for storing respondent pairwise judgement

Field name	Description
<b>id</b>	A key field. Storing the unique identity
<b>P01</b>	Store pairwise judgment for Population factor VS Administrative factor
<b>P02</b>	Store pairwise judgment for Population factor VS Transportation factor
<b>P03</b>	Store pairwise judgment for Population factor VS Landuse factor
<b>P04</b>	Store pairwise judgment for Population factor VS Student Flow factor
<b>P05</b>	Store pairwise judgement for Population factor VS Public Preference factor
<b>P06</b>	Store pairwise judgment for Administrative factor VS Transportation factor
<b>P07</b>	Store pairwise judgment for Administrative factor VS Landuse factor
<b>P08</b>	Store pairwise judgment for Administrative factor VS Student Flow factor
<b>P09</b>	Store pairwise judgment for Administrative factor VS Public Preference factor
<b>P10</b>	Store pairwise judgment for Transportation factor VS Landuse factor
<b>P11</b>	Store pairwise judgment for Transportation factor VS Student Flow factor
<b>P12</b>	Store pairwise judgement for Transportation factor VS Public Preference factor
<b>P13</b>	Store pairwise judgment for Landuse factor VS Student Flow factor
<b>P14</b>	Store pairwise judgment for Landuse factor VS Public Preference factor
<b>P15</b>	Store pairwise judgment for Student Flow factor VS Public Preference factor

After the saving process, the AHP calculation is conducted. Figure 1 is a line chart with the X-axis representing the consistency ratio and the Y-axis representing the percent number of the respondent's judgment result, which had corresponded range consistency ratio. There were just 5% of respondents with a consistency ratio from 0 to 0.1, and there were 35% of respondents with the consistency ratio from 0.1 to 0.2. Saaty said that if consistency ration was under 10% (0.1), the inconsistency could be accepted. However, he also noted that the researcher could increase the limit for higher number of pairwise. In this research, we used 20% or 0.2. However, although it rose to 0.2, still only 40% (5%+35%) judgments could be accepted. Therefore, this research decided to conduct another survey with a different method.

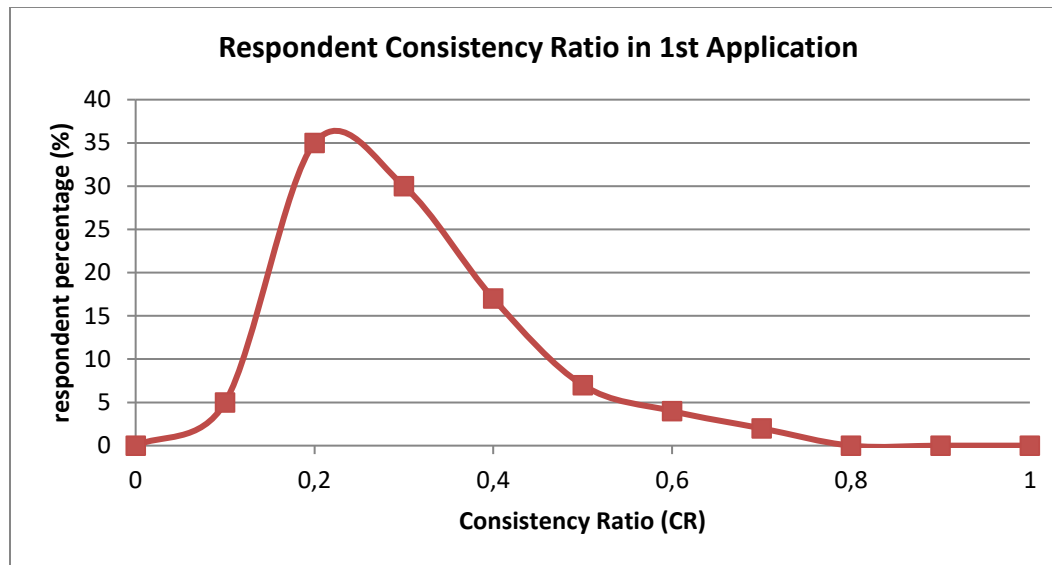


FIGURE 1. The consistency ratio statistic of the first AHP application

### DIRECTED PAIRWISE JUDGEMENT

The first MCDA application got a lack number of accepted judgments. The second MCDA application added a guide to direct the respondent to get a more consistent result. The form will limit the options in the pairing process. It will only show the options (radio buttons) that did not lead to inconsistency. The options in certain pairwise depended on the other pairwise. For example, pairwise options of B to C relies on the comparison of A to B, and A to C. If A to B had comparison point 1\_9 (B was extremely more important than A, see Table 2) and A to C had comparison point 9\_1 (A was extremely more important than C), then B to C should have comparison point 9\_1 (B was extremely more important than C). The ideal number for keeping the consistency should be 18\_1, but it was out of range; therefore, it could not be used. Table 2 shows the comparison point dependent matrix of B to C if A to B and A to C was known.

Some cells in the matrix have two or three pairwise points. For example, the cell of A to B has a comparison point of 9\_1, and A to C has a comparison point of 9\_1. The perfect consistency comparison point for B to C is 1\_1 because both B and C have the same comparison point with A. Here, the cell is not only filled with 1\_1 but also with 1\_3 and 3\_1. The purpose of this addition was to make the user still have choices but not lead to a big inconsistency range. This table describes the relationship between three factors: factor A, factor B, and Factor C. The next step replaced A, B, and C with the real factors. Table 3 shows the real factors and their dependents

Several first appeared pairwise still had full options: P01 to P05. All options (comparison points) of these pairwise were enabled because they had no previous affected pairwise. In P06, there were pairwise that affected the availability of the pairwise comparison points, which Starting referred to table 3 above. In P06, F2 to F3 comparison point depended on P01 (F1 to F2) and P02 (F1 to F3). Pairwise P07, P08, and P09 applied a similar situation. Starting in Pairwise P10, there are two or more conditions of the previous pairwise group that affected options composition. P10 is pairwise of F3 and F4. This pairwise option composition depends on two pairwise groups: F1-F3 F1-F4 and F2-F3 F2-F4. These two group results were joined with the OR operation. For example, if the enable options from F1-F3 and F1-F4 were 9\_1 and 7\_1 and the enable options from F2-F3 and F2-F4 were 7\_1 and 5\_1, the combination of these result consisted of 9\_1, 7\_1, and 5\_1. P11 and P12 had a similar characteristic. P13 and P14 had three dependents on pairwise groups, and P15 had four.

The respondents fill the survey by picking the appropriate option that matches their opinion. They could do it in sequence or at random. The composition of the options (enable and disable) in the pairwise was changed regularly during the survey. When the respondent has filled a pairwise, an action to calculate option composition in other pairwise was conducted. This action is described in the Pairwise action matrix in Table 4 below. This matrix shows what work should be performed after the respondent filling the pairwise comparison. For example, in pairwise P12, after respondent judging the pairwise in the P12 (F3-F6), the application checked it to ensure that there had been already a judgment for P10 (F3-F4) then took action to adjust P14 (F4-F6) and if it already had a judgment for P11 (F3-F5), the application would adjust P15 (F5-F6).

TABLE 2. The Comparison point dependent matrix

B-C?		A-B								
		9_1	7_1	5_1	3_1	1_1	1_3	1_5	1_7	1_9
A-C	9_1	1_1, 3_1, 1_3	1_1, 3_1	5_1, 3_1	7_1, 5_1	9_1, 7_1	9_1	9_1	9_1	9_1
	7_1	1_1, 1_3	1_1, 3_1, 1_3	1_1, 3_1	5_1, 3_1	7_1, 5_1	9_1, 7_1	9_1	9_1	9_1
	5_1	1_5, 1_3	1_1, 1_3	1_1, 3_1, 1_3	1_1, 3_1	5_1, 3_1	7_1, 5_1	9_1, 7_1	9_1	9_1
	3_1	1_7, 1_5	1_5, 1_3	1_1, 1_3	1_1, 3_1, 1_3	1_1, 3_1	5_1, 3_1	7_1, 5_1	9_1, 7_1	9_1
	1_1	1_9, 1_7	1_7, 1_5	1_5, 1_3	1_1, 1_3	1_1, 3_1, 1_3	1_1, 1_3	5_1, 3_1	7_1, 5_1	9_1, 7_1
	1_3	1_9	1_9, 1_7	1_7, 1_5	1_5, 1_3	1_1, 3_1	1_1, 3_1	1_1, 1_3	5_1, 3_1	7_1, 5_1
	1_5	1_9	1_9	1_9, 1_7	1_7, 1_5	1_5, 1_3	1_1, 3_1	1_1, 3_1	1_1, 1_3	5_1, 3_1
	1_7	1_9	1_9	1_9	1_9, 1_7	1_7, 1_5	1_5, 1_3	1_1, 3_1	1_1, 1_3	1_1, 1_3
	1_9	1_9	1_9	1_9	1_9	1_9, 1_7	1_7, 1_5	1_5, 1_3	1_1, 3_1	1_1, 1_3

TABLE 3. Pairwise dependent matrix

Pairwise	1 <sup>st</sup> Depend on	2 <sup>nd</sup> Depend on	3 <sup>rd</sup> Depend on	4 <sup>th</sup> Depend on 4
F1-F2 (P01)				
F1-F3 (P02)				
F1-F4 (P03)				
F1-F5 (P04)				
F1-F6 (P05)				
F2-F3 (P06)	A=F1, B=F2, C=F3 (P01, P02)			
F2-F4 (P07)	A=F1, B=F2, C=F4 (P01, P03)			
F2-F5 (P08)	A=F1, B=F2, C=F5 (P01, P04)			
F2-F6 (P09)	A=F1, B=F2, C=F6 (P01, P05)			
F3-F4 (P10)	A=F1, B=F3, C=F4 (P02,P03)	A=F2, B=F3, C=F4 (P06, P07)		
F3-F5 (P11)	A=F1, B=F3, C=F5 (P02,P04)	A=F2, B=F3, C=F5 (P06, P08)		
F3-F6 (P12)	A=F1, B=F3, C=F6 (P02,P05)	A=F2, B=F3, C=F6 (P06, P09)		
F4-F5 (P13)	A=F1, B=F4, C=F5 (P03,P04)	A=F2, B=F4, C=F5 (P07,P08)	A=F3, B=F4, C=F5 (P10, P11)	
F4-F6 (P14)	A=F1, B=F4, C=F6 (P03,P05)	A=F2, B=F4, C=F6 (P07,P09)	A=F3, B=F4, C=F6 (P10, P12)	
F5-F6 (P15)	A=F1, B=F5, C=F6 (P04,P05)	A=F2, B=F5, C=F6 (P08,P09)	A=F3, B=F5, C=F6 (P11,P12)	A=F4, B=F5, C=F6 (P13, P14)

TABLE 4. Pairwise Action matrix

Pairwise	Action1	action2	action3	action4
P01	if(P02) adjust(P06)	if(P03) adjust(P07)	if(P04) adjust(P08)	if(P05) adjust(P09)
P02	if(P01) adjust(P06)	if(P03) adjust(P10)	if(P04) adjust(P11)	if(P05) adjust(P12)
P03	if(P01) adjust(P07)	if(P02) adjust(P10)	if(P04) adjust(P13)	if(P05) adjust(P14)
P04	if(P01) adjust(P08)	if(P02) adjust(P11)	if(P03) adjust(P13)	if(P05) adjust(P15)
P05	if(P01) adjust(P09)	if(P02) adjust(P12)	if(P03) adjust(P14)	if(P04) adjust(P15)
P06	if(P07) adjust(P10)	if(P08) adjust(P11)	if(P09) adjust(P12)	
P07	if(P06) adjust(P10)	if(P08) adjust(P13)	if(P09) adjust(P14)	
P08	if(P06) adjust(P11)	if(P07) adjust(P13)	if(P09) adjust(P15)	
P09	if(P06) adjust(P12)	if(P07) adjust(P14)	if(P08) adjust(P15)	
P10	if(P11) adjust(P13)	if(P12) adjust(P14)		
P11	if(P10) adjust(P13)	if(P12) adjust(P15)		
P12	if(P10) adjust(P14)	if(P11) adjust(P15)		
P13	if(P14) adjust(P15)			
P14	if(P13) adjust(P15)			

The second MCDA Application was running for about nine months, starting in October 2017, and collected 147 respondent judgments. The invited respondents from the old MCDA application were re-asked for using the second application and added with other invited respondents. The result was calculated in a similar way as the first one. This result still sometimes generated some inconsistency in the ratio, but the number of inconsistency judgments was lower. It got 65 judgments with consistency ratio 0.1 29 judgments with consistency ratio 0.2, and just 6 judgments with consistency ratio 0.3. If the consistency ratio is 0.2, it got 94% accepted judgments. The number is more than twice the result of the first AHP application. Figure 2 shows the comparison of the respondent consistency ratio between the first and the second application.

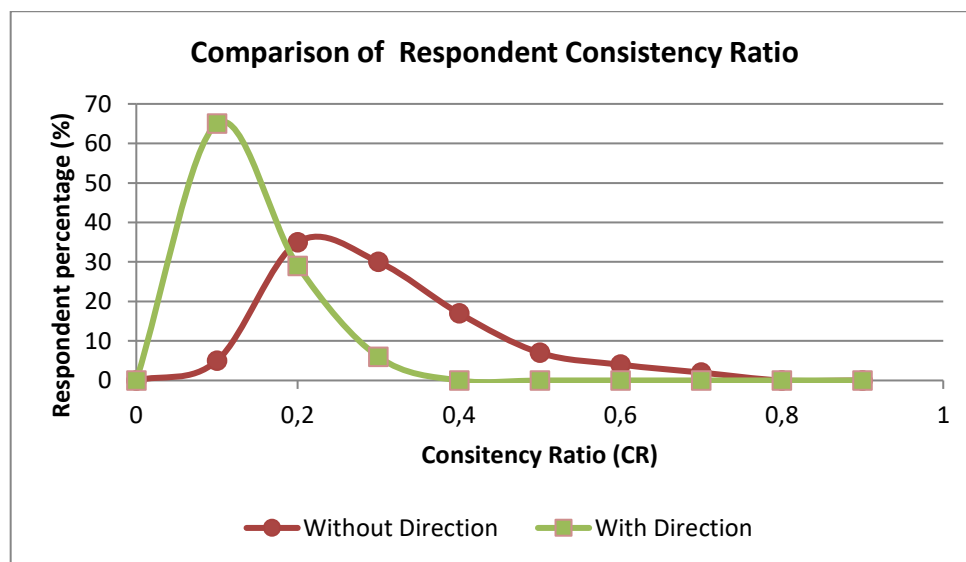


FIGURE 2. Comparison of Respondent Consistency ratio between the first application (without direction) and the second application (with direction)

## CALCULATING THE COMPLETION TIME

The first and the second AHP application were not forcing the judges to get the consistent pairwise comparison. The research which focuses on the AHP application continued to test the reliability of the directed pairwise judgment method. After it brought a significant number than the undirected pairwise judgment, the writer wants to know is it also shortens the comparison time if the judgment is required to be consistent. The second AHP application then updated with feature:

- A choice for the respondent to use the pairwise comparison page with direction or without direction. If the respondent chooses with direction, a method in the previous chapter will be adapted, else the comparison page will act as the first AHP application, and therefore all radio buttons are available.
- After the user completes all pairwise comparison, the application will calculate the consistency ratio
- Users will get a notice of whether the comparison is consistent or not. If not consistent, they will be asked to review the comparison until it was consistent.
- There is a timekeeper that starts when the user opens the pairwise comparison page and stops when the comparison is consistent ( $CR < 0.2$ ). When it happens, the pairwise comparison value is stored in the database.

Another survey was conducted. Start from November 2018 until October 2019, and it got 97 respondents choose not using the direction and 121 respondents using the direction. The application records how much time is needed for the respondent to end the comparison with accepted consistency. Figure 3 shows the time-consumption comparison between pairwise comparison with direction and pairwise comparison without the direction. The fastest time recorded in the pairwise comparison with the direction is about 40 seconds, and the slowest one is 75 seconds. The fastest time recorded in the pairwise comparison without the direction is about 55 seconds, and the slowest one is 120 seconds. Respondent in the pairwise comparison with the direction most often was completing the judgment about 60 seconds and in the pairwise comparison without the direction were 80 seconds. The average time for the pairwise comparison with the direction was 56 seconds and without direction were 81 seconds.

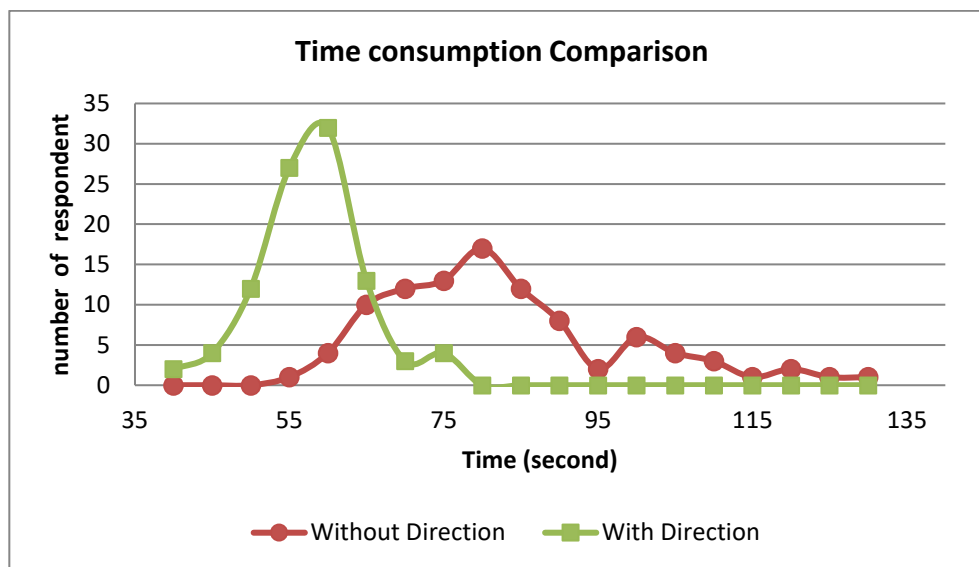


FIGURE 3. Time-consumption comparison chart between the pairwise comparison with direction and the pairwise comparison without direction



## CONCLUSION

This paper is a part of the main research with title A GIS-based Multi-Criteria Decision Analysis and Public Participatory Approach for Public Schools Site Selection and focused on the AHP pairwise judgment phase. This research encounters a 'wasting time' when almost a year collecting opinions got a unexpected result. The inconsistent pairwise comparison proportion was two times the consistent one. It happened because the researcher has no experience with managing AHP pairwise judgment before. Not all judges would care about consistency, especially if we didn't tell beforehand.

There are two kinds of processes that can resolve the inconsistency of pairwise comparison. The first is fixing the result by modifying the pairwise judgment to reduce inconsistency. The second is to recall the judges to fix their comparison. This research chooses the second one. The AHP application built with a new approach called 'directed pairwise judgment.' This application dynamically calculated the available options in the pairwise based on the previous pairwise judgments. This approach prevented the respondent from comparing the next pairwise with inconsistent value. The result of this second MCDA got a much higher number of respondent's judgment with an accepted consistency ratio than did the first one. The application was then updated with feedback and force the respondent for making the comparison until it got a particular consistency ratio. The algorithm in the second AHP application has also proven to provide a faster time for the user to get this specific consistency ratio. It can be concluded that the use of directed pairwise judgment approach and consistency ratio feedback generate consistent pairwise comparison with less time needed by the respondent.

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