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Glocal Control for Mecanum-Wheeled Vehicle with Slip Compensation

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***Abstract*— This paper designs a hierarchical decentralized controller for a mecanum-wheeled vehicle that is considered as a heterogeneous multi-agent system, according to the concept of glocal control. The equations of motion of each individual mecanum wheel and the entire vehicle with slip is analyzed to produce an interconnected, nonlinear model that can be linearized to apply linear control theory. The global objective is the motion of the vehicle, in terms of both position and orientation, produced by the effective forces of each wheel. The local objective is the driving force and slip control of each wheel, considering the interconnection between all agents. The proposed hierarchical LQR (linear quadratic regulator) controller ensures the satisfaction of both global and local objectives. Simulation results of a mecanum wheel are shown that verifies the performance of the method.**

Keywords— Mecanum wheel, glocal control, decentralized hierarchical control, slip control

I. INTRODUCTION

Nowadays, autonomous mobile robots are widely used in many industrial and household applications. These robots are typically non-holonomic systems that have limited maneuverability in tight, confined workspaces due to the minimum steering angle. This may necessitate multiple readjustments of the orientation to navigate through narrow pathways and around corners. In these situations, holonomic or omni-directional wheeled robots would provide better performance and maneuverability [1]. This includes mobile robots that employ mecanum wheels with the ability to move in any direction without changing the orientation of the vehicle.

However, mecanum-wheeled robots become difficult to control effectively when moving over surfaces with very low or very high coefficient of friction, such as an oily floor or rough concrete. In these conditions, undesired phenomenon such as slipping or skidding can be observed due to the reduced traction of mecanum wheels when compared with conventional wheels. Several kinematic and dynamic models have been developed for the mecanum wheel [2], and while these researches mentioned the effect of wheel traction, slip is generally omitted. For conventional non-holonomic robots, different methods have been proposed to control the slip ratio of the vehicle. A control system for driving force distribution of in-wheel motor electric vehicles was developed to examine the effect of acceleration on split-friction surfaces and utilizes glocal control with a decentralized hierarchical structure [3].

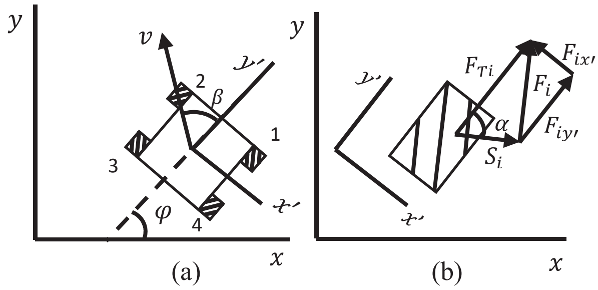

Fig. 1. a) Mecanum-wheeled vehicle model, b) driving and slip forces developed on a roller of the i^{th} wheel.

This paper investigates the dynamic model of each individual mecanum wheel and equations of motion of the vehicle with slip in order to establish a decentralized hierarchical structure. The upper layer consists of the global objective of position and orientation control of the vehicle. The lower layer is used for driving force control of each wheel. The interconnection between layers is determined and a glocal control strategy based on hierarchical LQR is proposed. Since the direction of motion of the vehicle depends on the coordination between all mecanum wheels, each driven by independent motors and subjected to different slip ratios, the concept of glocal control will be applied to achieve consensus between all agents and improve performance. The validity of the proposed controller is shown by simulation of a four-wheel mecanum robot.

II. MODELLING

A. Fundamental Vehicle Dynamics and Slip Ratio

This paper studies a mecanum-wheeled vehicle of total mass m with N in-wheel motors (IWMs), where N is assumed to be 4, according to [4]. The axis of rotation for the rollers makes an angle of α to the wheel rotation axis, where $\alpha = 45^o$. From Fig.1, let (x, y, z) refer to the stationary coordinate axis, while (x', y', z') refer to the body-attached coordinate axis at the geometrical center of the vehicle and i is the wheel number, φ is the orientation of the vehicle and β is the direction of the vehicle velocity. $\boldsymbol{F}_{Ti}$ is the force developed on the roller due to the motor torque T_i at the circumference of the i^{th} mecanum wheel. The developed torque can be written as:

$$T_i = r_i F_{Ti} = r_i \mu_d F_{Ni} + I_\omega \dot{\omega}_i \quad (1)$$

where ω_i and $\dot{\omega}_i$ is rotational velocity and acceleration of the i^{th} mecanum wheel respectively, and r_i is the radius of the i^{th} wheel, μ_d is the coefficient of dynamic friction between wheel and ground while the coefficient of viscous friction between the motor shaft and its bearings is neglected for simplicity, F_{Ni} is the frictional force proportional to the weight of the vehicle and I_ω is the inertia constant of the wheel about its center of mass.

The developed force $\boldsymbol{F}_{Ti}$ is divided into slip force $\boldsymbol{S}_i$, which is an ineffective force, and effective driving force $\boldsymbol{F}_i$. The sum of all the forces developed at all the wheels determines the motion of the vehicle. From Fig.1, $\boldsymbol{F}_i$ can be expressed as follows:

$$\boldsymbol{F}_i = \boldsymbol{F}_{Ti} sin\alpha \quad (2)$$

The direction of the resultant force from all the wheels is the same as the direction of the resultant driving velocity on the vehicle.

Then the slip ratio of a wheel is defined as $\boldsymbol{\lambda_i}$ which is divided into 2 components along both x and y-axis i.e. $\boldsymbol{\lambda_i} = [\lambda_{ix} \ \lambda_{iy}]^T$.

B. Driving Force Dynamics

For the purpose of control, the first-order dynamic model of wheel force is defined [3]:

$$\boldsymbol{\tau_i} \dot{\boldsymbol{F}}_i + \boldsymbol{F}_i = \overline{\boldsymbol{F}}_i \quad (3)$$

where $\boldsymbol{\tau_i}$ is the relaxation time constant that can be identified from $\boldsymbol{F}_{Ni}$ vs $\boldsymbol{\lambda_i}$ graph, $\boldsymbol{F}_i$ is the dynamic tire force and $\overline{\boldsymbol{F}}_i$ is the steady-state tire force where $\overline{\boldsymbol{F}}_i = \boldsymbol{S_{ti}} \boldsymbol{\lambda_i} + \boldsymbol{b_i}$ which $\boldsymbol{S_{ti}}$ is the driving stiffness and $\boldsymbol{b_i}$ is the y-intercept of the graph of μ_d vs $\boldsymbol{\lambda_i}$ [3]. These parameters are separated into x and y component in world frame i.e. $\boldsymbol{F}_i = \begin{bmatrix} F_{i_x} & F_{i_y} \end{bmatrix}^T$ and $\boldsymbol{\tau_i} = \begin{bmatrix} \tau_{i_x} & 0 \\ 0 & \tau_{i_y} \end{bmatrix}$.

By problem formulation, the following nonlinear system is established for the driving force and slip dynamics:

$$\dot{x}_i(t) = A_{ii}(t) x_i(t) + B_{ii} u_i(t) + \sum_{j=1}^{N} A_{ij}(t)\, x_j(t) + f_i(t)$$

$$y_i(t) = C_{ii} x_i(t) \quad (4)$$

$$x_i(t) = [\boldsymbol{F}_i(t) \ \boldsymbol{\lambda_i}(t)]^T$$
$$u_i(t) = T_i(t)$$
$$x_j(t) = \begin{bmatrix} \boldsymbol{F}_j(t) & \boldsymbol{\lambda_j}(t) \end{bmatrix}^T$$

$$A_{ii}(t) = \begin{bmatrix} -\frac{1}{\tau_{i_x}} & 0 & \frac{S_{ti_x}}{\tau_{i_x}} & 0 \\ 0 & -\frac{1}{\tau_{i_y}} & 0 & \frac{S_{ti_y}}{\tau_{i_y}} \\ a_{ii_{31}} & 0 & a_{ii_{33}} & 0 \\ 0 & a_{ii_{42}} & 0 & a_{ii_{44}} \end{bmatrix}, B_{ii}(t) = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\omega_i I_\omega} \\ \frac{1}{\omega_i I_\omega} \end{bmatrix}$$

$$C_{ii}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_{ij}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_{ij_{31}} & 0 & 0 & 0 \\ 0 & a_{ij_{42}} & 0 & 0 \end{bmatrix}$$

$$f_i(t) = \begin{bmatrix} \frac{b_{i_x}}{\tau_{i_x}} & \frac{b_{i_y}}{\tau_{i_y}} & \frac{\cot(\alpha + (-1)^i \varphi)\dot{\varphi}}{(-1)^i} & \frac{\tan(\alpha + (-1)^i \varphi)\dot{\varphi}}{(-1)^{i+1}} \end{bmatrix}^T$$

$$a_{ii_{31}} = -\left(\frac{C_1}{\omega_i \sin(\alpha + (-1)^i \varphi)}\right)$$

$$a_{ii_{33}} = -\left(\frac{\dot{\omega}_i}{\omega_i} + \frac{\cot(\alpha + (-1)^i \varphi)\dot{\varphi}}{(-1)^i}\right)$$

$$a_{ii_{42}} = -\left(\frac{-(C_1)(-1)^i}{\omega_i \cos(\alpha + (-1)^i \varphi)}\right)$$

$$a_{ii_{44}} = -\left(\frac{\dot{\omega}_i}{\omega_i} + \frac{\tan(\alpha + (-1)^i \varphi)\dot{\varphi}}{(-1)^{i+1}}\right)$$

$$a_{ij_{31}} = -\left(\frac{C_2}{\omega_i \sin(\alpha + (-1)^i \varphi)}\right)$$

$$a_{ij_{42}} = -\left(\frac{-(C_2)(-1)^i}{\omega_i \cos(\alpha + (-1)^i \varphi)}\right)$$

where $c_1 = \frac{r_i sin\alpha}{I_\omega}$, $c_2 = \frac{1}{m r_i sin\alpha}$.

For the purposes of controller design, this system can be linearized using Jacobian linearization.

III. Glocal Control System

A. Grouping and layering

Using the mecanum-wheeled vehicle model in Fig.1, a hierarchical control configuration consisting of 2 layers is established where $W(i)$ refers to the i^{th} wheel. The Lower Layer (LL) is a set that includes all wheels: $W = \{W(i)\}, i \in [1, N]$. In the Upper Layer (UL), information from all wheels are combined to drive the vehicle. Utilizing this grouping and layering, the motion control objectives can be obtained hierarchically as follows.

1) UL: Position and Orientation Control

Position and orientation control of the vehicle with N motors. This layer determines the reference motion of the vehicle and sends the command signal to each wheel. It receives information from all the wheels and compute for orientation and position of the robot using (1).

2) LL: Driving Force and Wheel Angular Velocity Control

Motor torque command T_i is generated for tracking the actual driving force with the reference value $\boldsymbol{F}_i^*$ by controlling the linearized system (4) and the driving force and wheel angular velocity of the i^{th} wheel can be computed by (1).

IV. Hierarchical LQR Design

A. Problem setting

Consider a mecanum-wheeled vehicle having $N = 4$ wheels, where the i^{th} wheel is an agent obtained from linearizing (4) as:

$$\dot{x}_i = a_i x_i + b_i x_i + \sum_{j=1}^{N} a_{ij}\, x_j \qquad (5)$$
$$y_i = c_i x_i$$

where $a_i, a_{ij} \in \mathbb{R}^{n_i \times n_i}$, $b_i \in \mathbb{R}^{n_i \times m_i}$, $c_i \in \mathbb{R}^{p_i \times n_i}$, which $0 < m_i < n_i$ and $i = 1, \cdots, N$. $x_i, x_j \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, and $y_i \in \mathbb{R}^{p_i}$ are the state vector of the i^{th} agent, the input vector of the i^{th} agent, and the output of the i^{th} agent respectively. The subsystem of the i^{th} wheel is denoted as $H_i(s)$.

The hierarchical network model of the vehicle is:

$$\dot{x} = \mathcal{A}x + \mathcal{B}u \qquad (6)$$
$$y = \mathcal{C}x$$

where $x = [x_1^T, \cdots, x_N^T]^T \in \mathbb{R}^n$, $u = [u_1^T, \cdots, u_N^T]^T \in \mathbb{R}^m$, $y = [y_1^T, \cdots, y_N^T]^T \in \mathbb{R}^p$, $\mathcal{A} = diag\{a_k\}_{k=1,\cdots,N} + (\Gamma \odot a_{ij}) \in \mathbb{R}^{n \times n}$, $\mathcal{B} = diag\{b_k\}_{k=1,\cdots,N} \in \mathbb{R}^{n \times m}$, $\mathcal{C} = diag\{c_k\}_{k=1,\cdots,N} \in \mathbb{R}^{p \times n}$ and $\Gamma \in \mathbb{R}^{N \times N}$, which Γ is inter-layer interaction matrix. Note that $diag\{(\cdot)_k\}_{k=1,\cdots,N}$ is diagonal matrix of $(\cdot)_1$to $(\cdot)_N$ and $\odot$ refers to Khatri Rao Product.

Consider the graph $\mathcal{G}$ that represents the information structure K consisting of weights K_{ij} among N wheels shown in Fig. 2. Nodes and edges ε represent each wheel and the interconnection between two wheels respectively. K_{ij} is set as follow:

$$K_{ij} = \begin{cases} 1, & \text{when } i \neq j \text{ and } (i,j) \in \varepsilon \\ \sum_{j=1}^{N} K_{ij}, & \text{when } i = j \end{cases} \qquad (7)$$

The information exchange is made by each wheel sending out aggregate signals z_i and z_{p_i} to cooperate with other connected wheels to realize the global objectives. Simultaneously, each wheel receives the signals w_i and w_{p_i} sent by other connected wheels individually.

Each subsystem $G_i(s)$ is implemented with its individual local controller giving $u_{\ell,i}$ as an output as shown in Fig. 3 having the control input:

$$u_i = w_i + w_{p_i} + u_{\ell,i} \qquad (8)$$

Then the control input for the whole hierarchical network with two layers is represented by:

$$u = w + w_p + u_\ell = (K \otimes I_{m_i})z + (\Gamma \otimes I_{m_i})z_p + u_\ell \qquad (9)$$

where $w = [w_1^T, \cdots, w_N^T]^T$, $w_p = \left[w_{p_1}^T, \cdots, w_{p_N}^T\right]^T$, $z = [z_1^T, \cdots, z_N^T]^T$, $z_p = \left[z_{p_1}^T, \cdots, z_{p_N}^T\right]^T$, $u_\ell = \left[u_{\ell,1}^T, \cdots, u_{\ell,N}^T\right]^T$, $K \otimes I_{m_i}$ and $\Gamma \otimes I_{m_i}$ are information structure and physical interconnection respectively, and $\otimes$ is the Kronecker product.

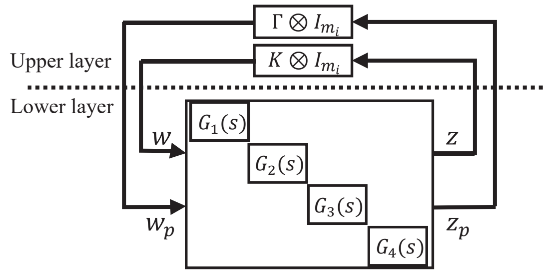

Fig. 2. Block diagram of hierarchical networked control system.

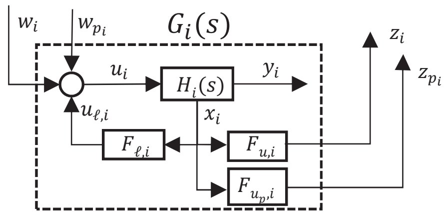

Fig. 3. Block diagram of state feedback controller.

B. Performance Indexes

In order ensure optimal performance, the following quadratic performance indexes have to be minimized.

$$J = J_x + J_u \qquad (10)$$
$$J_x = J_{x,\mathcal{L}} + J_{x,\mathcal{G}} \qquad (11)$$

where J_x relates to the local and global objectives and J_u is a penalty for the control input, $J_{x,\mathcal{L}}$ is a local performance index composing of the individual penalties for the states of subsystems, and $J_{x,\mathcal{G}}$ is a global performance index.

C. State Feedback Design Procedure

The following steps are implemented in order to design the hierarchical decentralized state-feedback controller following [5].

Step 1: Local LQR Design

Select the weighting matrices for the local objectives, $Q_i \in \mathbb{R}^{n_i \times n_i}$ and $R_i \in \mathbb{R}^{m_i \times m_i}$ such that $\left(Q_i^{1/2}, a_i\right)$ is observable and $R_i \succ 0$ for $i = 1, \cdots, N$. Then solve the corresponding local Riccati equation:

$$P_i a_i + a_i^T P_i - P_i b_i R_i^{-1} b_i^T P_i + Q_i = 0 \qquad (12)$$

for the unique positive definite solution $P_i \in \mathbb{R}^{n_i \times n_i}$.

Step 2: Global LQR Design

Set K following to (7) and the weighting matrices $R_g \in \mathbb{R}^{m \times m}$, $R_{p_g} \in \mathbb{R}^{m \times m}$, $Q_g \in \mathbb{R}^{n \times n}$and $Q_{p_g} \in \mathbb{R}^{n \times n}$ for $J_{x,\mathcal{G}}$ as follows:

$$R_g = r_{g1}(\mathbf{1}_N \mathbf{1}_N^T) \otimes I_{m_i} + r_{g2} I_m \qquad (13)$$
$$R_{p_g} = r_{p_g 1}(\mathbf{1}_N \mathbf{1}_N^T) \otimes I_{m_i} + r_{p_g 2} I_m \qquad (14)$$
$$Q_g = P_\ell \mathcal{B} R_g \mathcal{B}^T P_\ell \qquad (15)$$
$$Q_{p_g} = P_\ell \mathcal{B} R_{p_g} \mathcal{B}^T P_\ell - P_\ell (I_N \odot a_{ij}) - (I_N \odot a_{ij}^T) P_\ell \qquad (16)$$

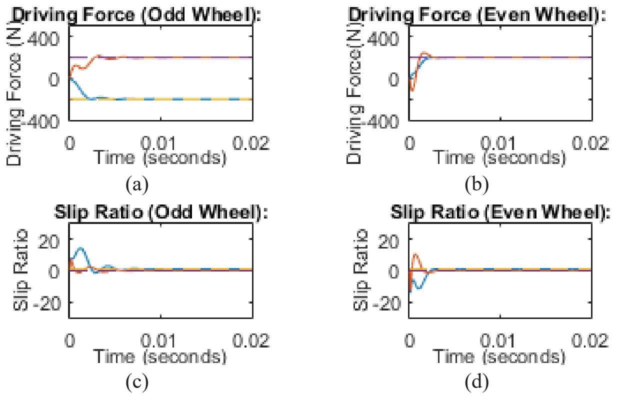

Fig. 4. Response of the wheel $\left(Q_{odd} = 10^6 * I_4, R_{odd} = 10, Q_{even} = 10^7 * I_4, R_{even} = 10, r_{g1} = r_{p_g 2} = 0.01, r_{g2} = r_{p_g 2} = 0.01\right)$

Where $r_{g1} > 0, r_{g2} > 0, r_{p_g 1} > 0, r_{p_g 2} > 0,$ $P_\ell = diag\{P_i\}_{i=1,\cdots,N} \succ 0$

Note that $\mathbf{1}_N = [1 \cdots 1] \in \mathbb{R}^{1\times N}$.

Step 3: State Feedback Gain Calculation

Set the state feedback gains $F_{\ell,i}$, $F_{u,i}$ and $F_{p_u,i}$ as follows:

$$F_{\ell,i} = -(R_{\ell,i} + r_{p_g 2}\Gamma_{ii} I_{m_i} + r_{g2} K_{ii} I_{m_i}) \mathcal{B}_i^T P_i \quad (17)$$
$$F_{u,i} = -r_{g1} \mathcal{B}_i^T P_i \quad (18)$$
$$F_{p_u,i} = -r_{p_g 1} \mathcal{B}_i^T P_i \quad (19)$$

V. SIMULATION RESULT

This section shows the simulation result of the proposed hierarchical LQR controller with state feedback with reference trajectory applied to control the driving force and slip of mecanum wheels. The initial driving force and slip in (4) are set as $[0 \quad 0 \quad 0 \quad 0]^T$ for all wheel. The objective is to control these values to $[-200 \quad 200 \quad 1 \quad 0]^T$ and $[200 \quad 200 \quad 1 \quad 0]^T$ for odd$(i = 1,3)$ and even$(i = 2,4)$ wheel respectively. The results are shown in Fig. 4 and Fig. 5 where I_4 is identity matrix of size 4x4, $(\cdot)_{odd} = (\cdot)_1 = (\cdot)_3$, and $(\cdot)_{even} = (\cdot)_2 = (\cdot)_4$. Appropriate gain parameters have to be chosen carefully to get such a response as Fig.4 (a)-(d). The inappropriate values of parameters give such a result shown in Fig.5 (a)-(d). Red solid line is the actual y-component of each parameter. Blue solid line is the actual x-component of each parameter. Purple dotted line is the reference y-component of each parameter. Yellow dotted line is the reference x-component of each parameter. From Fig. 4 (a), rise time and maximum overshoot of driving force response in x-direction of odd wheel are 2.2108 ms and 1.993% while rise time and maximum overshoot of driving force response in y-direction of odd wheel are 2.7212 ms and 7.314%. From Fig. 4 (b), rise time and maximum overshoot of driving force response in x-direction of even wheel are 2.4656 ms and 0.197% while rise time and maximum overshoot of driving force response in y-direction of even wheel are 1.1227 ms and 22.494%. From Fig. 5 (a), rise time and maximum overshoot of driving force response in x-direction of odd wheel are 1.1396 ms and 89.56% while rise time and maximum overshoot of driving force response in y-direction of odd wheel are 0.5063 ms and 57.97%. From Fig. 5 (b), rise time and maximum overshoot of driving force response in x-direction of even wheel are 1.5514 ms and 4.565% while rise time and maximum overshoot of driving force response in y-direction of even wheel are 8.8114 ms and 149.674%

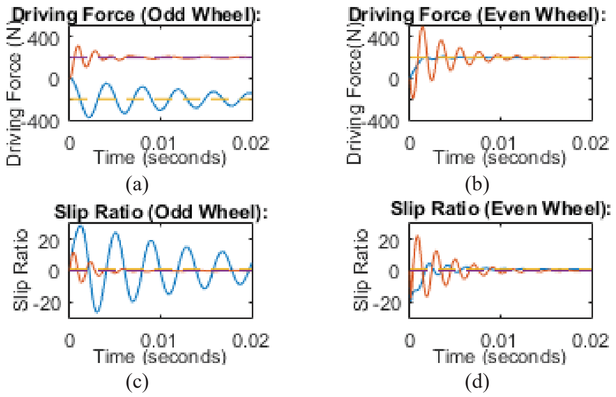

Fig. 5. Response of the wheel $\left(Q_{odd} = 10 * I_4, R_{odd} = 10, Q_{even} = 10^5 * I_4, R_{even} = 10, r_{g1} = r_{p_g 1} = 0.01, r_{g2} = r_{p_g 2} = 0.01\right)$

VI. CONCLUSION

In this paper, a controller is developed for a mecanum-wheeled vehicle. The control system utilizes a decentralized hierarchical structure according to the glocal control concept. Firstly, an interconnected, nonlinear model is developed for each wheel and the entire vehicle. This model is linearized. Secondly, the heterogeneous agents are grouped together to define two layers corresponding to global and local objectives. Lastly, a hierarchical LQR controller is designed to achieve driving force and slip control of all interconnected mecanum wheels.

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