PROCEEDINGS

APCOMS

The 1st Asia Pacific Conference on Manufacturing Systems

The 8th National Conference on Production Systems

A Challenge for Collaborative Manufacturing Systems

Ramada Bintang Bali Resort
Kuta-Bali, Indonesia
September 5-6, 2007

PRODUCTION SYSTEM LABORATORY
MANUFACTURING SYSTEM RESEARCH GROUP
DEPARTMENT OF INDUSTRIAL ENGINEERING
FACULTY OF INDUSTRIAL TECHNOLOGY
INSTITUT TEKNOLOGI BANDUNG

ISSN:

CONTENTS

Integrated Production-Distribution Planning with Considering Preventive Maintenance

Amelia Santoso
Industrial Engineering Department
Institut Teknologi Bandung, Bandung 40132, INDONESIA
Industrial Engineering Department, Universitas Surabaya, Surabaya 60292, INDONESIA
+62-22-251 0680, Email: amelia@ubaya.ac.id

Senator Nur Bahagia and Suprayogi
Industrial Engineering Department, Institut Teknologi Bandung, Bandung 40132, INDONESIA
+62-22-251 0680, Email: {senator, yogi}@mail.ti.itb.ac.id

Dwiwahju Sasongko
Chemical Engineering Department Institut
Teknologi Bandung, Bandung 40132, INDONESIA
+62-22-250 4551, Email: sasongko@che.itb.ac.id

Abstract. The preventive maintenance activity is important thing in production system especially for a continuous production process, for example in fertilizer industry. Therefore, it has to be considered in production-distribution planning. This paper considers the interval of production facility's preventive maintenance in production-distribution planning of multi echelon supply chain system which consists of a manufacturer with a continuous production process, a distribution center, a number of distributors and a number of retailers. The problem address in this paper is how to determine coordinated production-distribution policies that considers the interval of production facility's preventive maintenance, and customer demand only occurred at retailers and it fluctuates by time. Based on model of Santoso, et al. (2007), using the periodic review inventory model and a coordinated production and replenishment policies that are decided by central planning office and it must be obeyed by all entities of multi-echelon supply chain, the integrated production-distribution planning model is developed to determine the production and replenishment policies of all echelon in the supply chain system in order to minimize total system cost during planning horizon. Total system cost consists of set-up/ordering cost, maintenance cost, holding cost, outsourcing cost and transportation cost at all of entities. With considering preventive maintenance and there is one production run over the planning horizon, the replenishment cycle at distribution center, distributors and retailers that are found out are greater than the basic model. Also, the multiplication of replenishment cycle at distribution center in production cycle that is found out is greater than the basic model but the multiplication of replenishment cycle at retailers in its distributor are smaller than the basic model.

Keywords: production-distribution planning, preventive maintenance, continuous production, time dependent demand

1. INTRODUCTION

Although some manufacturer use a continuous production process, such as in the fertilizer and paper manufacturers, researches in the integrated production-distribution planning mainly address the discrete production process, such as Chen and Chen (2005), Lin and Lin (2005), Weng (2004), Routroy and Kodali (2005), Nur Bahagia and Toruan (2001), and Nur Bahagia and Sofitra (2001). This paper proposed integrated production-distribution model that consider a continuous production process.

Generally, the reliability of production facility will deteriorate as its age. The unreliable production facility could cause the facility breakdown. The production will be stopped until the facility has already repaired. The breakdown of the facility certainly bothers the production schedule. For minimizing this condition, the preventive maintenance is needed by manufacturer especially for manufacturer using continuous production process. Although considered a continuous production process, Santoso *et al.* (2007) has not considered preventive maintenance in their integrated production-distribution model. This paper considers preventive maintenance when we determine production and distribution policy.

The demand pattern considered in previous researches can be deterministic, such as Chen and Chen (2005), Jolayemi and Olorunniwo (2004) and Haq *et al.* (1991); probabilistic, such as Lin and Lin (2005), Weng (2004), Routroy and Kodali (2005), Nur Bahagia and Toruan (2001), and Nur Bahagia and Sofitra (2001); and time-dependent, such as Santoso *et al.* (2007). In this paper, time-dependent demand will be considered as well as continuous production process and preventive maintenance when we develop the integrated production-distribution model.

This paper determines optimal production time at the manufacturer, replenishment cycle of distribution center, distributors and retailers, integer of a multiplication of the replenishment cycle at the manufacturer, distribution center and distributors in order to minimize the total system cost during planning horizon.

2. FRAMEWORK

This integrated production-distribution model is developed for 4-echelons supply chain systems that consists a manufacturer, a distribution center, a number of distributors and a number of retailers as is illustrated in the framework in Figure 1.

Because of using a continuous process, the manufacturer produces the products continuously until the preventive maintenance schedule of the production facility or all demand during planning horizon had been produced. The length of planning horizon is determined from interval of preventive maintenance and time consumption for maintaining the production facility. In other side, the customer demand that is faced by retailers fluctuates by time (time dependent demand). The manufacturer supplies all retailers through distribution center and distributor.

In order to minimize the total system cost, there are the need of deciding a coordinated production and replenishment policies between the manufacturer, the distribution center, the distributors and the retailers.

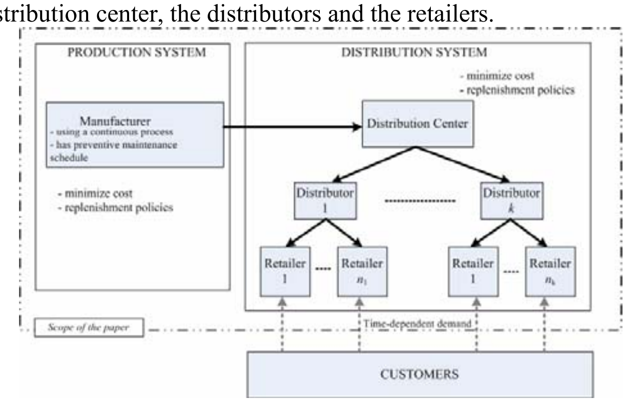

Figure 1: Framework

In this paper, the integrated production-distribution model is developed based on the previous model that was developed by Santoso *et al.* (2007). Similar with Santoso *et al.*(2007), this research uses coordinated policy, echelon inventory concept, and single cycle time policy as the approaches in developing the model.

The coordinated policy approach leads the manufacturer, the distribution center, the distributors and the retailers to perform coordination for determining the production and the replenishment policies. The production and the replenishment policies that have been decided must be obeyed by all entities in the supply chain systems.

Principally, Clark and Scarf (1960) stated that in the echelon inventory concepts, total inventory at an entity in one echelon is the sum of on-hand and in-transit inventories and all inventories at its downstream echelons. This concept is very different with the normal calculation of holding cost that uses average inventory at each entity.

Using the single cycle time policy (Nur Bahagia, 1999 and Abdul-Jalbar *et al*., 2006) means that all entities start to produce or order product in the same time. As a result, the planning horizon (T) would be an integer multiplied by the replenishment cycle of distribution center (Tg), i.e., T=$NgTg$, where Ng is an integer. By the same token, the replenishment cycle of distribution center (Tg) is an integer multiplied by the replenishment cycle of distributor k (Td_k) and the replenishment cycle of distributor k (Td_k) is also an integer multiplied by the replenishment cycle of retailer j; $j \in k$ (Tr_{kj}). Generally, the single cycle time policy can be formulated as follow:

$$T = N_g T_g = N_g N_{d_k} T_{d_k} = N_g N_{d_k} N_{r_{kj}} T_{r_{kj}} \tag{1}$$

where:

Ng an integer of a multiplication of the replenishment cycle of the distribution center in a production cycle

Nd_k an integer of a multiplication of the replenishment cycle of distributor *k* in a replenishment cycle of the distribution center

Nr_{kj} an integer of a multiplication of the replenishment cycle of retailer *j* in a replenishment cycle of distributor *k*; $j \in k$

3. MODEL DEVELOPMENT

The integrated production-distribution model is developed for determining production and distribution policy in order to minimize total system cost per planning horizon. The total system cost per planning horizon (*C*) consists of total cost per planning horizon at the manufacturer (*Co*), distribution center (*Cg*), distributors (*Cd*) and retailers (*Cr*).

Using a continuous process for producing the product at manufacturer implies the production is run continuously until the time to maintain the facility or all the customer demands during the planning horizon are fulfilled. That means in a planning horizon, there is only one production run. The planning horizon (*T*) is a summation of preventive maintenance interval (*wt*) and time consumption for maintaining machine (*wp*).

$$T = w_t + w_p \quad (2)$$

The assumptions used in developing the model are:

- The customer's demand at all retailers depend on time and are deterministic
- No stock-outs are permitted at all echelons
- Manufacturer can supply all of the distribution center's demand from its production and outsourcing
- Outsourcing product at the manufacturer is allowed only if the production capacity is not enough to satisfy demand
- Unused production capacity is utilized for satisfying another kind of demands or exported demand.
- The production facility is never failure before the schedule of preventive maintenance
- Raw material are always available
- The entire replenishment lot size is added to inventory at the same time (for distribution center, all distributors and all retailers)
- Products at a distributor can not be transferred to other distributors; similarly at a retailer.
- Production capacity is large enough to supply all customer demands.
- Production cost per unit; ordering cost at distribution centers, distributors and retailers; and transportation cost per unit are constant

Due to the pattern of the customer demand at retailer is deterministic time-dependent, inventory at the end of cycle is zero and safety stock is not necessary. Therefore the level of maximum inventory per period is equal to total demand per period. Because the pattern of demand is time-dependent so the amount of customer demand per period is not equal and finally it causes the level of maximum inventory is not equal for each period. Alike with Santoso et al. (2007), in this model, a polynomial function is used as an approximation for the time-dependent pattern of the customer demand.

Total cost of retailer per planning horizon consists of replenishment cost and holding cost per planning horizon at all retailer *j*, $j \in k$. The replenishment cost is established from replenishment frequency during planning horizon and cost per replenishment while the holding cost is calculated from average of inventory per planning horizon and carrying cost per unit per planning horizon. Hence, total cost of retailer per planning horizon is formulated as follows:

$$C_r = \sum_{\forall k} \sum_{\forall j \in k} \left\{ \frac{A_{r_{kj}} T}{T_{r_{kj}}} + \frac{H_{r_{kj}}}{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} \int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\, dt \right\} \quad (3)$$

where:

Ar_{kj} cost of each product replenishment at retailer *j* that is supplied by distributor *k*

Tr_{kj} replenishment cycle of retailer *j* supplied by distributor *k*

Hr_{kj} product holding cost per ton per year at retailer *j* that is supplied by distributor *k*

zr_{kj} demand function at retailer *j*; $j \in k$ (a polynomial function of time)

A distributor distributes the product to several retailers that are geographically dispersed. Therefore, transportation cost is a cost component of distributor's total cost per planning horizon as well as replenishment cost and holding cost. The first, replenishment cost is obtained from multiplication of replenishment frequency during a planning horizon and cost per replenishment. The next cost component is holding cost that is obtained from average of inventory per planning horizon at distributor and carrying cost per unit per planning horizon. As result of using echelon inventory concept, total inventory at distributor k is the total inventory at distributor k itself and total inventory at all retailer j that are supplied by distributor k. The last cost component is transportation cost that is determined by amount of product deliveries per planning horizon and transport cost per unit from distributor k to retailer j; $j \in k$. According to the explanation in above and total demand at a distributor is the summation of total demand at all retailers that are supplied by that distributor, the total cost of distributor per planning horizon can be formulated as follows:

$$C_d = \sum_{\forall k} \left\{ \frac{A_{d_k} T}{T_{d_k}} + \frac{H_{d_k}}{N_g N_{d_k} T_{d_k}} \sum_{\forall j \in k} \left(\int_0^{N_g N_{d_k} T_{d_k}} z_{r_{kj}}(t)\,dt + \int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt \right) + \sum_{j \in k} C_{r_j}^{d_k} \int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt \right\} \quad (4)$$

where:

Td_k replenishment cycle of distributor k

Ad_k cost of each product replenishment at distributor k

Hd_k product holding cost per ton per year at distributor k

$C_{r_j}^{d_k}$ transportation cost of product from distributor k to retailer j; $\forall j \in k$

Similar with distributor, there are three cost components in the distribution center: replenishment cost, holding cost and transportation cost. First, the replenishment cost per planning horizon is determined by replenishment frequency during planning horizon and cost per replenishment. The next cost component, the holding cost per planning horizon is equal to the multiplication of inventory average at distribution center during planning horizon and carrying cost per unit per planning horizon. Based on echelon inventory concept, total inventory at distribution center is the sum of total inventory at distribution center itself and total inventory at all of its downstream echelons. The last cost component is the transportation cost which is determined by amount of product deliveries during planning horizon and transport cost per unit from distribution center to all distributors. Similarly with the way of the total demand calculation at a distributor, total demand at distribution center is the sum of total demand at all distributors that are supplied by distribution center. Derived from the explanation in above, total cost per planning horizon at distribution center can be illustrated as follows:

$$C_g = \frac{A_g T}{T_g} + \frac{H_g}{N_g T_g} \sum_{\forall k} \sum_{j \in k} \left(\int_0^{N_g T_g} z_{r_{kj}}(t)\,dt + \int_0^{N_g N_{d_k} T_{d_k}} z_{r_{kj}}(t)\,dt + \int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt \right) + \sum_{\forall k} C_{d_k}^{g} \int_0^{N_g N_{d_k} T_{d_k}} z_{d_k}(t)\,dt \quad (5)$$

where:

Tg replenishment cycle of distribution center

Ag cost of each product replenishment at the distribution center

Hg product holding cost per ton per year at the distribution center

zd_k demand function at distributor k

$C_{d_k}^{g}$ transportation cost of product from distribution center to distributor k

At the manufacturer, production is run continuously until the time to maintain the production facility or all demands during the planning horizon are fulfilled. If the production is stopped because the time to maintain the facility before the products produced are enough to satisfy all demand, the manufacturer will outsource the lack of demand and production. During a planning horizon, there is only a preventive maintenance schedule of production facility so the planning horizon is determined by the sum of preventive maintenance interval and the consumption time for maintaining the facility. According to these situations, total cost per planning horizon at manufacturer consists of setup cost, maintenance cost, outsourcing cost, holding cost and transportation cost.

There is only one production cycle during a planning horizon so that the setup cost per planning horizon is equal to cost per setup. Also, there is only one preventive maintenance schedule during a planning horizon so that the maintenance cost is equal to cost for maintaining production facility. The consequence of considering the interval of preventive maintenance in determining production and distribution policy, outsourcing cost is one of cost component in total cost of manufacturer. The outsourcing cost is determined by total demand that can not be satisfied from production during planning horizon and outsourcing cost per unit. In other side, the holding cost is calculated using average of inventory at manufacturer during planning horizon and carrying cost per unit per planning horizon. According to echelon inventory concept, total inventory at manufacturer is equal to the sum of total inventory at manufacturer itself and at all its downstream echelons. The transportation cost is equal to amount of product deliveries at manufacturer during planning horizon multiply with transport cost per unit from manufacturer to distribution center. In addition to consider total demand at manufacturer is equal to total demand at distribution center, total cost per planning horizon at manufacturer can be derived as follows:

$$C_o = A_o + A_p + P_{os}\max\left(0, \sum_{\forall k}\sum_{j\in k}\int_{w_t}^{BTS} z_{r_{kj}}(t)\,dt\right) + \frac{H_o}{T}\sum_{\forall k}\sum_{j\in k}\left(\int_0^{T} z_{r_{kj}}(t)\,dt - \int_0^{t_p} z_{r_{kj}}(t)\,dt + \int_0^{N_gT_g} z_{r_{kj}}(t)\,dt + \int_0^{N_g N_{d_k}T_{d_k}} z_{r_{kj}}(t)\,dt + \int_0^{N_g N_{d_k}N_{r_{kj}}T_{r_{kj}}} z_{r_{kj}}(t)\,dt\right) + C_g^o\int_0^{N_gT_g} z_g(t)\,dt$$

with

$$BTS = \frac{\sum_{\forall k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt}{\psi} \tag{6}$$

where:

tp	optimal production duration for satisfying demand during planning horizon
Ap	cost of production facility maintenance
Ao	production facility setup cost
Ho	product holding cost per ton per year at the manufacturer
Pos	product outsourcing cost per ton
ψ	production rate
wt	interval of production preventive maintenance
wp	time consumption for maintaining machine
z_g	demand function at distribution center
C_g^o	transportation cost of product from manufacturer to distribution center

Finally, the objective function can be derived. The objective function is total system cost per planning horizon that comprises of the total cost per planning horizon at manufacturer, distribution center, distributors and retailers, as in the following formula:

Minimize the total system cost

$$C = A_o + A_p + P_{os}\max\left(0, \sum_{\forall k}\sum_{j\in k}\int_{w_t}^{BTS} z_{r_{kj}}(t)\,dt\right) + \frac{H_o}{T}\sum_{\forall k}\sum_{j\in k}\left(\int_0^{T} z_{r_{kj}}(t)\,dt - \int_0^{t_p} z_{r_{kj}}(t)\,dt + \int_0^{N_gT_g} z_{r_{kj}}(t)\,dt + \int_0^{N_g N_{d_k}T_{d_k}} z_{r_{kj}}(t)\,dt + \int_0^{N_g N_{d_k}N_{r_{kj}}T_{r_{kj}}} z_{r_{kj}}(t)\,dt\right) +$$

$$C_g^o\int_0^{N_gT_g} z_g(t)\,dt + \frac{A_gT}{T_g} + \frac{H_g}{N_gT_g}\sum_{\forall k}\sum_{j\in k}\left(\int_0^{N_gT_g} z_{r_{kj}}(t)\,dt + \int_0^{N_gN_{d_k}T_{d_k}} z_{r_{kj}}(t)\,dt + \int_0^{N_gN_{d_k}N_{r_{kj}}T_{r_{kj}}} z_{r_{kj}}(t)\,dt\right) + \sum_{\forall k} C_{d_k}^g \int_0^{N_gN_{d_k}T_{d_k}} z_{d_k}(t)\,dt + \sum_{\forall k}\left\{\frac{A_{d_k}T}{T_{d_k}} + \frac{H_{d_k}}{N_gN_{d_k}T_{d_k}}\sum_{\forall j\in k}\left(\int_0^{N_gN_{d_k}T_{d_k}} z_{r_{kj}}(t)\,dt + \int_0^{N_gN_{d_k}N_{r_{kj}}T_{r_{kj}}} z_{r_{kj}}(t)\,dt\right) + \sum_{\forall j\in k} C_{r_j}^{d_k}\int_0^{N_gN_{d_k}Nr_{kj}Tr_{kj}} z_{r_{kj}}(t)\,dt\right\} + \sum_{\forall k}\sum_{\forall j\in k}\left\{\frac{A_{r_{kj}}T}{T_{r_{kj}}} + \frac{H_{r_{kj}}}{N_gN_{d_k}N_{r_{kj}}T_{r_{kj}}}\int_0^{N_gN_{d_k}N_{r_{kj}}T_{r_{kj}}} z_{r_{kj}}(t)\,dt\right\}$$

with

$$BTS = \frac{\sum_{\forall k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt}{\psi} \tag{7}$$

Subject to the following constraints:

$$T_o = N_gT_g = N_gN_{d_k}T_{d_k} = N_gN_{d_k}N_{r_{kj}}T_{r_{kj}} \tag{8}$$

$$\int_0^1 z_o(t)\,dt = \int_0^1 z_g(t)\,dt = \sum_{\forall k}\int_0^1 z_{d_k}(t)\,dt = \sum_{\forall k}\sum_{\forall j\in k}\int_0^1 z_{r_{kj}}(t)\,dt \tag{9}$$

$$T_g \le T - t_p \tag{10}$$

$$t_p = \begin{cases} \dfrac{\sum_{\forall k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt}{\psi} & \text{if } \dfrac{\sum_{\forall k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt}{\psi} < T - w_p \\ w_t & \text{if } \dfrac{\sum_{\forall k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt}{\psi} \ge T - w_p \end{cases} \tag{11}$$

Ng, Nd_k, $Nr_{kj} \ge 1$ and integer value (12)

Constraint (8) pertains to use the single cycle time policy approach explained in the framework. Constraint (9) ensure that the total annual demand at manufacturer is the same as the total annual demand at distribution center, and also is the same as the total annual demand at all distributors and is the same as the total annual demand at all retailers. The next constraint (10) ensures that total demand at the distribution center especially in the last replenishment cycle should be fulfilled. Constraint (11) guarantee there is no production when the production facility is maintained. And the last constraint (12) ensure decision variables have to be integer and always greater than zero.

Constraint (11) can be simplified become

$$t_p = \min\left[\left(\frac{1}{\psi}\sum_{\forall k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt\right), w_t\right] \tag{13}$$

After simplification, together with constraint (8) and (9), constraint (13) is substituted to equation (7) so that the objective function will become:

Minimize the total system cost

$$C = A_o + A_p + \frac{H_o}{T}\sum_{\forall k}\sum_{j\in k}\left(\int_0^T z_{r_{kj}}(t)\,dt - \int_0^{t_p} z_{r_{kj}}(t)\,dt + 3\int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt\right) +$$
$$P_{os}\max\left(0, \sum_{\forall k}\sum_{j\in k}\int_{w_t}^{BTS} z_{r_{kj}}(t)\,dt\right) + C_g^o\sum_{\forall k}\sum_{j\in k}\int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt + A_g N_g +$$
$$\sum_{\forall k}\sum_{j\in k}\frac{3H_g}{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}}\int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt + \sum_{\forall k}\sum_{j\in k} C_{d_k}^g \int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt +$$
$$\sum_{\forall k}\left\{A_{d_k} N_g N_{d_k} + \sum_{\forall j\in k}\frac{2H_{d_k}}{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}}\int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt +\right.$$
$$\left.\sum_{\forall j\in k} C_{r_j}^{d_k}\int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt\right\} + \sum_{\forall k}\sum_{\forall j\in k}\left\{A_{r_{kj}} N_g N_{d_k} N_{r_{kj}} +\right.$$
$$\left.\frac{H_{r_{kj}}}{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}}\int_0^{N_g N_{d_k} N_{r_{kj}} T_{r_{kj}}} z_{r_{kj}}(t)\,dt\right\}$$

with

$$BTS = \frac{\sum_{\forall k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt}{\psi} \quad \text{and} \quad t_p = \min\left[\left(\frac{1}{\psi}\sum_{\forall k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt\right), w_t\right] \tag{14}$$

Subject to the following constraints:

$$N_g\left(T - \min\left[\left(\frac{1}{\psi}\sum_{\forall k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt\right), w_t\right]\right) \geq T \tag{15}$$

$Ng, Nd_k, Nr_{kj} \geq 1$ and integer value (16)

Decision variables Ng, Nd_k and Nr_{kj} in the model that is developed, should be integer so that the model can be solved using branch and bound method. This method is composed of three steps: bounding, branching and fathoming.

After the decision variables Ng, Nd_k, Nr_{kj} and Tr_{kj} are found out, the level of maximum inventory for each replenishment cycle at the manufacturer, distribution center, distributors and retailers can be found out too. The formulas for obtaining the level of maximum inventory at all echelons are in the following:

At the manufacturer:

$$R_o = \sum_{k}\sum_{j\in k}\int_0^T z_{r_{kj}}(t)\,dt \tag{17}$$

At the distribution center:

$$R_{g_p} = \sum_{k}\sum_{j\in k}\int_{(p-1)T_g}^{pT_g} z_{r_{kj}}(t)\,dt \qquad p = 1,\ldots,N_g \tag{18}$$

At distributor k:

$$R_{d_{kl}} = \sum_{j\in k}\int_{(l-1)T_{d_k}}^{lT_{d_k}} z_{r_{kj}}(t)\,dt \qquad l=1,\ldots, N_g Nd_k;\ \forall k \tag{19}$$

At the retailer j:

$$R_{r_{kjs}} = \int_{(s-1)T_{r_{kj}}}^{sT_{r_{kj}}} z_{r_{kj}}(t)\,dt \qquad s=1,\ldots, N_g Nd_k Nr_{kj};\ \forall j,k \tag{20}$$

where:

- s replenishment cycle index at retailer j during planning horizon
- l replenishment cycle index at distributor k during planning horizon
- p replenishment cycle index at distribution center during planning horizon
- Ro level of maximum inventory at the manufacturer
- Rg_p level of maximum inventory in the pth replenishment cycle at the distribution center
- Rd_{kl} level of maximum inventory in the lth replenishment cycle at distributor k
- Rr_{kjs} level of maximum inventory in the sth replenishment cycle at retailer j; $j\in k$

4. NUMERICAL EXAMPLE

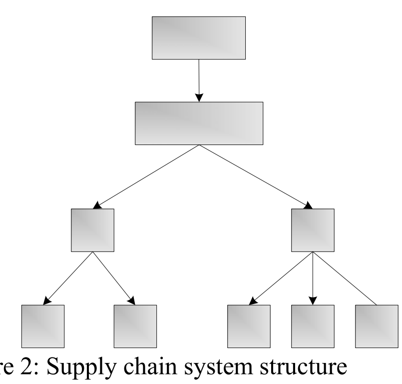

Figure 2: Supply chain system structure

The model will be applied using the following numerical example. The supply chain system consists of a manufacturer with a continuous production process, a distribution center, two distributors and five retailers as shown in Figure 2. Retailer R1 and R2 are supplied by distributor D1, while retailer R3, R4 and R5 are supplied by distributor D2.

The customer demand at each retailer that has time-dependent pattern and identical, is approximated using a polynomial function.

$$z_{r_{kj}}(t) = 82{,}590t^4 - 144{,}900t^3 + 79{,}830t^2 - 16{,}690t + 2{,}885$$

Table 1: Transport cost per unit

Origin point	Destination point	Transport cost per unit (rupiahs)
Manufacturer	Distribution center	100
Distribution center	Distributor 1	60
	Distributor 2	70
Distributor 1	Retailer 1	40
	Retailer 2	35
Distributor 2	Retailer 3	25
	Retailer 4	30
	Retailer 5	40

Table 2. Setup or replenishment cost, maintenance cost and holding cost for each scenario

		Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Production capacity (ton)		8,000	8,000	8,000	7,000	8,000
Cost per setup or replenishment (rupiah)	Manufacturer: Setup cost (A_o)	6,500,000	650,000	6,500,000	6,500,000	650,000
	Manufacturer: Maintenance cost (A_p)	6,000,000	600,000	6,000,000	6,000,000	600,000
	Distribution center (A_g)	4,000,000	40,000	400,000	400,000	80,000
	Distributor 1 (Ad_1)	4,500,000	450,000	4,500,000	4,500,000	4,500,000
	Distributor 2 (Ad_2)	4,000,000	40,000	4,000,000	4,000,000	4,000,000
	Retailer 1 (Ar_{11})	100,000	100,000	10,000,000	10,000,000	100,000
	Retailer 2 (Ar_{12})	110,000	110,000	11,000,000	11,000,000	110,000
	Retailer 3 (Ar_{21})	125,000	125,000	12,500,000	12,500,000	125,000
	Retailer (Ar_{22})	120,000	120,000	12,000,000	12,000,000	120,000
	Retailer 5 (Ar_{23})	120,000	120,000	12,000,000	12,000,000	120,000
Carrying cost per ton per year (rupiah)	Manufacturer (H_o)	149,200	149,200	149,200	149,200	149,200
	Distribution center (H_g)	165,700	16,570,000	165,700	165,700	2,657,000
	Distributor 1 (Hd_1)	183,600	1,836,000	183,600	183,600	183,600
	Distributor 2 (Hd_2)	183,000	1,830,000	183,000	183,000	183,000
	Retailer 1 (Hr_{11})	195,000	195,000	195,000	195,000	195,000
	Retailer 2 (Hr_{12})	195,000	195,000	195,000	195,000	195,000
	Retailer 3 (Hr_{21})	195,500	195,500	195,500	195,500	195,500
	Retailer (Hr_{22})	194,000	194,000	194,000	194,000	194,000
	Retailer 5 (Hr_{23})	195,500	195,500	195,500	195,500	195,500
Outsourcing cost per ton (rupiah)	Manufacturer (Pos)	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000

The consequence of using coordinated planning, total demand at distributor k is equal to the sum of total demand at all retailers that are supplied by distributor k. Total demand at distribution center is equal to total demand at all distributor. Because there is only one distribution center that are supplied by manufacturer so that total demand at the manufacturer is equal to total demand at distribution center.

The production facility will be maintained periodically, every 0.8 year. Total time consumption for maintaining the production facility is 0.1 year. Therefore, the planning horizon is 0.9 year. Moreover, transport cost from certain echelon to its downstream echelon is shown in Table 1.

Table 3. Ratio of cost per setup or replenishment to carrying cost per ton

	Ratio of cost per setup or replenishment to carrying cost per ton				
	scenario 1	scenario 2	scenario 3	scenario 4	scenario 5
Manufacturer	83.78016	8.37802	83.78016	83.78016	8.37802
Distribution center	24.14001	0.00241	2.41400	2.41400	0.03011
Distributor 1	24.50980	0.24510	24.50980	24.50980	24.50980
Distributor 2	21.85792	0.02186	21.85792	21.85792	21.85792
Retailer 1	0.51282	0.51282	51.28205	51.28205	0.51282
Retailer 2	0.56410	0.56410	56.41026	56.41026	0.56410
Retailer 3	0.63939	0.63939	63.93862	63.93862	0.63939
Retailer 4	0.61856	0.61856	61.85567	61.85567	0.61856
Retailer 5	0.61381	0.61381	61.38107	61.38107	0.61381

Table 4. The optimal solution and total system cost

		scenario 1	scenario 2	scenario 3	scenario 4	scenario 5
Optimal production duration (tp)		0.76389	0.76389	0.76389	0.80000	0.76389
Production or replenishment cycle (year)	Manufacturer (T)	0.9	0.9	0.9	0.9	0.9
	Distribution center (Tg)	0.12857	0.09000	0.12857	0.10000	0.12857
	Distributor 1 (Td_1)	0.12857	0.09000	0.12857	0.10000	0.12857
	Distributor 2 (Td_2)	0.12857	0.09000	0.12857	0.10000	0.12857
	Retailer 1 (Tr_{11})	0.12857	0.09000	0.12857	0.10000	0.12857
	Retailer 2 (Tr_{12})	0.12857	0.09000	0.12857	0.10000	0.12857
	Retailer 3 (Tr_{21})	0.12857	0.09000	0.12857	0.10000	0.12857
	Retailer (Tr_{22})	0.12857	0.09000	0.12857	0.10000	0.12857
	Retailer (Tr_{23})	0.12857	0.09000	0.12857	0.10000	0.12857
Integer multiple	Distribution center (Ng)	7	10	7	9	7
	Distributor 1 (Nd_1)	1	1	1	1	1
	Distributor 2 (Nd_2)	1	1	1	1	1
	Retailer 1 (Nr_{11})	1	1	1	1	1
	Retailer 2 (Nr_{12})	1	1	1	1	1
	Retailer (Nr_{21})	1	1	1	1	1
	Retailer (Nr_{22})	1	1	1	1	1
	Retailer (Nr_{23})	1	1	1	1	1
Total system cost		10,386,208,395.44	366,854,804,140.36	10,825,874,071.72	11,075,823,366.89	548,216,892,582.10

Five scenarios are defined in this numerical example that include the setup or replenishment cost, maintenance cost, outsourcing cost and carrying cost as shown in Table 2. Ratio of cost per setup or replenishment to carrying cost is shown in Table 3.

Using branch and bounds methods, the optimal solution for all scenarios can be found out as shown in Table 4. The changing of production capacity and ratio of cost per setup or replenishment to carrying cost per ton affect the optimal solution whether production duration at manufacturer, integer of multiplication of replenishment cycle of the distribution center in a production cycle or replenishment cycle at distribution center, distributors and retailers.

According to optimal solution of scenario 1, 3 and 5 in Table 4, we can conclude that production duration, length and integer multiple of replenishment cycle do not change while ratio of cost per setup or replenishment to carrying cost per ton at distribution center varies between 0.03011 and 24.14. The optimal solution: integer multiplication of replenishment cycle of the distribution center in a production cycle and replenishment cycle at distribution center have just changed when ratio of cost per setup or replenishment to carrying cost per ton at distribution center is 0.00241. Thus, it implies that the changing of ratio of replenishment cost to carrying cost at distribution center only influences on the optimal solution at distribution center especially integer multiplication of replenishment cycle of the distribution center in a production cycle.

The consequence of considering the deterministic time-dependent demand is that the level of maximum inventory is not similar for each replenishment period. For instance, the level of maximum inventory at the manufacturer, distribution center, distributions and retailer for scenario 1 can be shown in Table 5.

The differences of this paper when it is compared with Santoso *et al.* (2007) are: considering interval of preventive maintenance in developing the model, there is only one production run over the planning horizon and the total demand during the planning horizon will be satisfied from outsourcing if the production capacity is not enough. The same value of setup cost, replenishment cost and carrying cost are used in scenario 1 of this paper and Santoso *et al*. (2007). The results of both can be seen at Table 6.

Table 5. The level of maximum inventory

Cycle	Level of maximum inventory				
	Manufac-turer	Distribution center	Distributor 1	Distributor 2	All retailers
1	6,111.20	1,401.0	560.44	840.65	280.22
2		1,112.40	444.97	667.45	222.48
3		1,173.90	469.55	704.33	234.78
4		1,094.00	437.60	656.40	218.80
5		729.54	291.82	437.72	145.91
6		285.43	114.17	171.26	57.09
7		314.80	125.92	188.88	62.96

Table 6. The comparison of result

		This paper	Santoso *et al.* (2007)
Production or replenish-ment cycle (year)	Manufacturer (T)	0.9	0.071
	Distribution center (Tg)	0.12857	0.071
	Distributor 1 (Td_1)	0.12857	0.071
	Distributor 2 (Td_2)	0.12857	0.071
	Retailer 1 (Tr_{11})	0.12857	0.0237
	Retailer 2 (Tr_{12})	0.12857	0.0237
	Retailer 3 (Tr_{21})	0.12857	0.0355
	Retailer 4 (Tr_{22})	0.12857	0.0355
	Retailer 5 (Tr_{23})	0.12857	0.0355
Integer multiple	Distribution center (Ng)	7	1
	Distributor 1 (Nd_1)	1	1
	Distributor 2 (Nd_2)	1	1
	Retailer 1 (Nr_{11})	1	3
	Retailer 2 (Nr_{12})	1	3
	Retailer (Nr_{21})	1	2
	Retailer (Nr_{22})	1	2
	Retailer (Nr_{23})	1	2

Based on Table 6, the replenishment cycle at distribution center, distributors and retailers in this paper are greater than Santoso *et al*. (2007). The production cycle at this paper is production duration of total demand over the planning horizon but in Santoso *et al*. (2007) is a production run and there may be more than one production run during a planning horizon. Moreover, in this paper, the multiplication of replenishment cycle at distribution center in production cycle is greater than Santoso *et al.* (2007) but the multiplication of replenishment cycle at retailers in its distributor are smaller than Santoso *et al.* (2007).

5. CONCLUSION

This paper has proposed an integrated production-distribution model for 4-echelon supply chain systems considered preventive maintenance interval. In this model, the manufacturer has a continuous production process and the retailers have a time-dependent demands pattern but identical for all retailers. The effect of considering time-dependent demand is that the level of maximum inventory is not equal for each replenishment period at the manufacturer, distribution center, distributors and retailers.

With considering preventive maintenance and there is one production run over the planning horizon, the replenishment cycle at distribution center, distributors and retailers in this paper are greater than Santoso *et al*. (2007). Also, in this paper, the multiplication of replenishment cycle at distribution center in production cycle is greater than Santoso *et al.* (2007) but the multiplication of replenishment cycle at retailers in its distributor are smaller than Santoso *et al.* (2007).

The proposed model is based on an assumption that no stock-out is permitted and the production capacity is large enough to supply all customer demands. Further research using a probabilistic time-dependent demand would contribute to our understanding of the behavior of this system. A supply rationing policy can be considered in this model as an extended model if the production capacity is less than customer demands.

REFERENCES

Abdul-Jalbar, B., Gutierrez, J.M., & Sicilia, J. (2006). Single cycle policies for the one-warehouse *N*-retailer inventory/distribution system. *Omega* **34**, 196-208

Chen, J.M. and Chen, T.H., (2005). The multi-item replenishment problem in a two-echelon supply chain: the effect of centralization versus decentralization. *Computers & Operations Research*, **32**, 3191-3207.

Clark, A.J., and Scarf, H. (1960). Optimal policies for a multi-echelon inventory problem. *Management Science*, **6**, 475-490.

Haq, A.N., Vrat, P. and Kanda, A., (1991). An integrated production-inventory-distribution model for manufacture of urea: a case. *International Journal of Production Economics*, **39**, 39-49.

Jolayemi, J.K. and Olorunniwo, F.O., (2004). A deterministic model for planning production quantities in a multi-plant, multi-warehouse environment with extensible capacities. *International Journal Production Economics*, **87**, 99-113.

Lin, C. and Lin, Y., (2005). The production size and inventory policy for a manufacturer in a two-echelon inventory model. *Computers & Operations Research*, **32**, 1181–1196.

Nur Bahagia, S., (1999). An optimization model of integrated 3-echelon value chain systems. *Proc. 4th Production System Conference*, ITB-Bandung (in Indonesian).

Nur Bahagia, S. and Sofitra, M., (2001). An integrated 3-echelon logistics system model by considering distribution route. *Jurnal Teknik dan Manajemen Industri ITB*, **21**(2), 1-19 (in Indonesian).

Nur Bahagia, S. and Toruan, J.L., (2001). Developing an optimization model of integrated 3-echelon value chain systems (a manufacturer, a depot and n retailers with direct and indirect supply). *Proc. 5th Production System Conf.*, ITB-Bandung (in Indonesian).

Routroy, S. and Kodali, R., (2005). Differential evolution algorithm for supply chain inventory planning. *Journal of Manufacturing Technology Management*, **16**(1), 7-17.

Santoso, A., Nur Bahagia, S., Suprayogi and Sasongko, D., (2007). Integrated production-distribution planning with time dependent demand in multi-echelon supply chain. *Proc. 2nd Operation and Supply Chain Management Conference*, Bangkok

Weng, Z.K. (2004). Coordinating order quantities between the manufacturer and the buyer: A generalized newsvendor model. *European Journal of Operational Research*, **156**, 148-161.

AUTHOR BIOGRAPHIES

A. Santoso has obtained her master degree from Department of Industrial Engineering, Institut Teknologi Bandung (ITB). Now, she is a PhD candidate at Department of Industrial Engineering, Institut Teknologi Bandung. Her research interest is in the area of supply chain management, especially in production-distribution system at multi echelon supply chain system.

S. Nur Bahagia is a Professor in the Department of Industrial Engineering, Institut Teknologi Bandung (ITB). Now, he is the head of Industrial Systems Planning and Optimization Laboratory, ITB and also, Research Group on Industrial System and Techno-Economics, ITB. He obtained his PhD degree in Distribution system from IAE Aix-en-Provence, Universite d'Aix - Marseille III, France. His research interests are in the area of logistics and supply chain management, mainly multi-echelon inventory and production-distribution systems.

Suprayogi is an Assistant Professor in the Faculty of Industrial Technology, Institut Teknologi Bandung (ITB). He belongs to Research Group on Industrial System and

Techno-Economics, ITB. He received his PhD degree in Engineering from The University of Tokyo, Japan. His research interests are in operations research, logistics and transportation.

D. Sasongko has been a faculty member in the Department of Chemical Engineering, Institut Teknologi Bandung (ITB) since 1980. And now, he is an Associate Professor in that department. Before joining the institute he worked for International Business Machines (IBM) for more than 4 years. He obtained his bachelor and master degrees in chemical engineering from ITB and earned a PhD degree in the same field from University of New South Wales, Australia. He has written more than 90 scientific papers published in national and international journals and proceedings. He has been serving as the Dean of Faculty of Industrial Technology ITB since 2004.